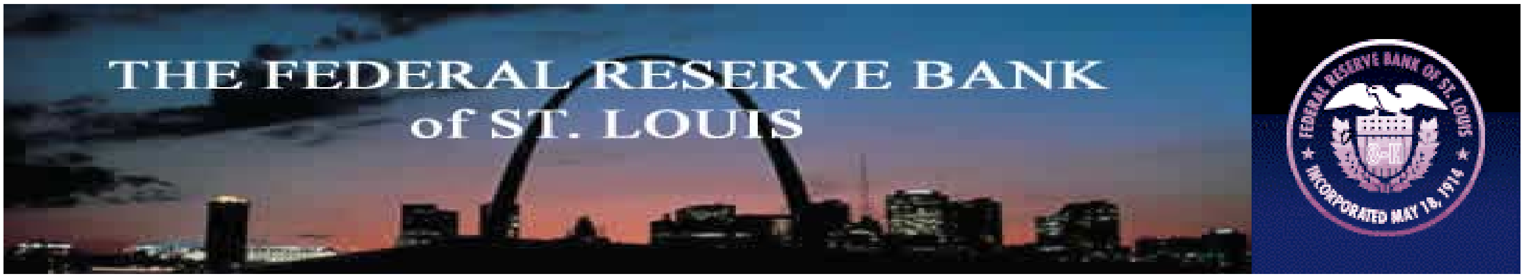

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Why Does Inventory Investment Fluctuate So Much During Contractions?

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Working Paper 1994-029A
http://research.stlouisfed.org/wp/1994/1994-029.pdf

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WHY DOES INVENTORY INVESTMENT FLUCTUATE SO MUCH DURING CONTRACTIONS?

ABSTRACT

Inventory investment appears to have a significant impact on the movement of aggregate output during business cycle contractions. Recent empirical evidence has raised doubts about the often used assumption of a buffer-stock/production-smoothing motivation for inventory. Work by Blinder and Maccini suggests that the use of an (S,s), or intermittent adjustment decision rule, better explains the stylized facts of the dynamics of inventory investment. This has led to the focus on the (S,s) as an alternative to production-smoothing. I assume that some agents use the (S,s) adjustment rule while others attempt to smooth production in the face of convex costs and uncertain demand. I simulate the interaction of heterogeneous agents (representing manufacturing, wholesale and retail agents) with different inventory decision rules to demonstrate that the stylized facts can be explained by a disaggregated model with vertical coupling between agents. The simulations find opposite aggregation bias effects for (S,s) agents than for production smoothing agents. In particular, aggregation horizontally across agents and/or temporally decreased the relative variability of production/ordering to sales for (S,s) agents while it increased the relative variability for production smoothing agents. The simulations also revealed synchronization by (S,s) agents when subjected to aggregate shocks. This may explain in some of the asymmetrical characteristics of the business cycle.

KEYWORDS: Simulation, Inventory Investment, Heterogeneous, Production Smoothing, (S,s), Aggregation, Business Cycles

JEL CLASSIFICATION: E22, E23, E32

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1. INTRODUCTION

The role of inventory investment in business cycle contractions has been well documented (Blinder and Maccini 1991). Explanations of aggregate inventory investment dynamics have been tempered by the need to reconcile firm level behaviour with the aggregate data. Metzler's (1941) seminal article established the potential for endogenous inventory cycles in a closed economy with lagged response of production to changes in demand. The models used by Metzler were *ad hoc* and served to illustrate the impact of varying lag coefficients. Since then, most of the inventory research has focussed on establishing a viable model of the firm's motivation for inventory and the resulting decision rules. The conventional wisdom, pioneered by Holt, Modigliani, Muth and Simon (1960), holds that firms use inventories as a buffer to smooth production in the face of uncertain demand and convex (quadratic) costs. The inventory investment decision rule used to accomplish smoothing is assumed to be a partial stock adjustment about a desired level of inventory. More recently it has been recognized that the partial adjustment or smoothing hypothesis is deficient in explaining the observed movement in aggregate inventory investment. The empirical analyses of aggregate data find higher variability of production than sales and a positive correlation between sales and inventory. The former appears to contradict the production smoothing motivation, and the latter appears to contradict the buffer stock motivation for holding inventory.

Work by Blinder and Maccini suggests that a model of intermittent adjustment or (S,s)[1] model does a better job of explaining the observed stylized facts of inventory investment movement than the production smoothing or buffer stock models. In particular, the observed higher variability of production than sales in aggregate manufacturing and trade data is more readily explained with an (S,s) model. The retail sector and the materials and supplies sector of manufacturing account for most of the variability in trade inventories (See Blinder 1990). These two sectors are likely candidates for an (S,s) decision rule because there are potential economies of scale due to fixed cost associated with ordering or delivery and possible volume discounts.

Intermittent adjustment models tend to be intractable at the aggregate level if agents are subject to both aggregate and idiosyncratic shocks. Path dependence and random perturbations limit general statements to a few highly restrictive scenarios. Work by Caplin (1985), Caballero and Engle (1991) and Bertola and Caballero (1990) is beginning to establish a framework for evaluating the aggregate dynamics of economies where agents make lumpy adjustments.

The typical measure of variability is the ratio of the variance of production to the variance of sales. A ratio less than 1.0 is assumed to confirm smoothing, while a ratio greater than 1.0 is assumed to

[1] The (S,s) inventory management model is characterized by a bandwidth of inventory with a lower limit designated as s and an upper limit designated as S. When inventory levels fall below s, the agent purchases enough to raise the inventory levels to S. When inventory lies within the band, no action is taken.

contradict smoothing. Lovell (1993) and Lai (1991) question the validity of using this variance ratio as a measure of production smoothing. Lovell's simulation measured variance ratio greater than 1.0 when agents were smoothing production. Lai shows how the relative size of aggregate and idiosyncratic shocks can result in variance ratios that exceed 1.0 when data are aggregated over agents who smooth production. The implication is that the observed aggregate stylized facts may not contradict the microeconomic production smoothing motivation; the apparent contradiction may in fact be due to the effects of aggregation.

This paper demonstrates that the existence of heterogeneous agents who face varying motivations and costs of holding inventory, in conjunction with the vertical linkage of these agents, can help explain the stylized facts observed in aggregate inventory dynamics. This vertical aggregation as well as horizontal and temporal aggregation can distort the variance ratio, supporting the argument that the variance ratio may not be a valid statistic to determine smoothing.

Due to the inherent difficulty in obtaining an analytic solution when agents use lumpy adjustments, the interaction of three levels of agents under a variety of production/order smoothing and lumpy (S,s) rules are simulated. To highlight some of the implications of horizontal aggregation, each sector is split into four identical firms. Each firm is subjected to idiosyncratic and/or aggregate shocks to sales. The resulting means and variances of production, sales and inventory are observed at the individual levels and in the aggregate to determine how well the stylized facts are explained by this model.

I find that when at least one set of agents within a vertical hierarchy uses an (S,s) decision rule for inventory management, variability in aggregate production appears more like (S,s) than smoothing. I also find that the presence of aggregate shocks can cause (S,s) agents to synchronize their purchases and behave like one large (S,s) agent, thus exacerbating the relative variability. As idiosyncratic shocks become more important, the relative variability of the aggregate tends to decrease due to less synchronization. The impact of aggregating the data temporally and horizontally across agents differs for production smoothing agents and (S,s) agents. Aggregation biased the variance ratio of (S,s) agents downwards and biased the variance ratio of production smoothers upwards.

These results suggest, on the one hand, that variance ratios of aggregate data that exceed 1.0 can reflect either aggregate production smoothing agents or aggregate (S,s) agents. On the other hand, the results suggest a potential method of identifying the underlying decision rule by disaggregating temporally or by agent. If the disaggregation results in a higher variance ratio, this would suggest an underlying dominant (S,s) rule; if the disaggregated data resulted in a lower variance ratio, this would suggest that the dominant rule was production smoothing.

The multi-sector model also sheds some light on the correlation between sales and inventory investment. Most scenarios do not result in the positive correlation between sales and inventory displayed in the stylized empirical facts. They did, however, demonstrate a tendency toward positive correlation when

(S,s) decisions were used, when production smoothers were facing sales from other production smoothers, or when (S,s) agents were aggregated temporally.

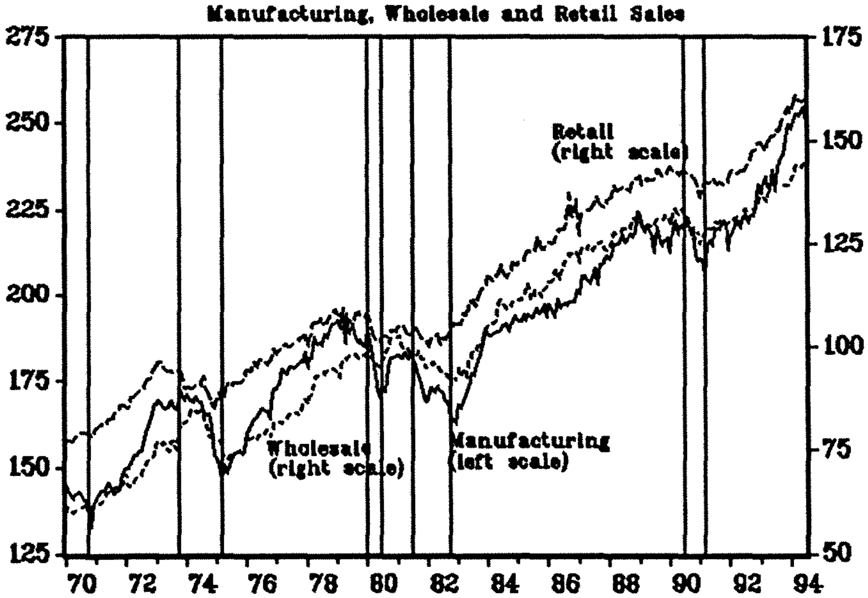
The paper is organized as follows. Section 2 discusses the historical movement of inventory investment and reviews the relevant literature and its relationship to this paper. Section 3 formalizes the proposed disaggregated model. Section 4 discusses implications of aggregation. Section 5 presents results and conclusions.

2. LITERATURE REVIEW

Inventory Investment and Recessions

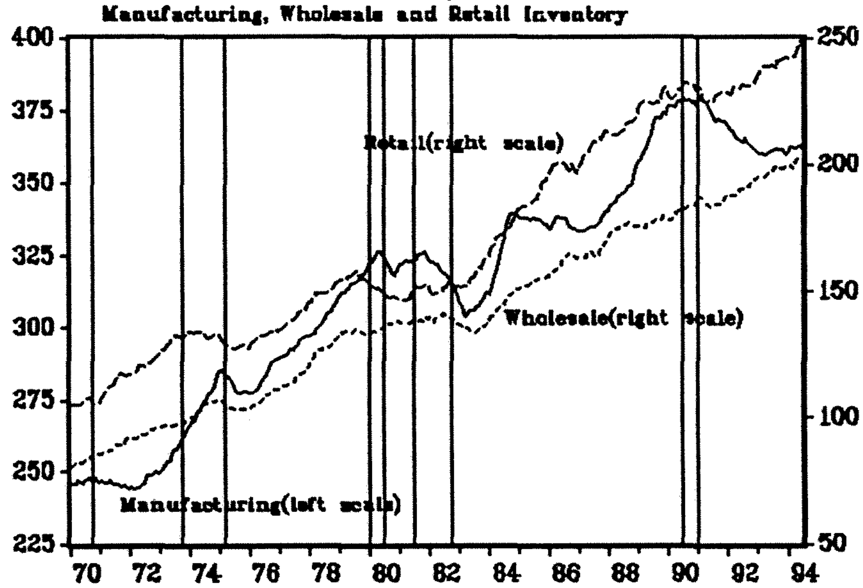
Historically, most post-war recessions have been characterized by a significant reduction in aggregate inventory investment relative to the reduction from peak to trough of the GNP. Figure 1 shows a plot of manufacturing, wholesale and retail sales from January, 1970 to July 1994, with the recessionary periods as defined by the NBER indicated by vertical lines. Figure 2 shows the associated inventory investment at the manufacturing, wholesale and retail levels for the same time period. The general comovement over the period is easily seen. During the recessionary periods bracketed by the vertical lines, there appears to be a negative correlation between the movement of sales and inventory in the initial months of the period.

Figure 1
Trade Sales by Sector 1970-94
Manufacturing, Wholesale and Retail Sales



Source: FAME
Seasonally Adjusted
Billions of 1987$

Figure 2
Trade Inventories by Sector 1970-94
Manufacturing, Wholesale and Retail Inventory



Source: FAME
Seasonally Adjusted
Billions of 1987$

Blinder (1990), evaluating data from the interwar and post-war recessions through 1980, observes that for the post-war period,

> "Inventory change has, on average, accounted for 101 percent of the peak to trough decline in real GNP; or keeping score a different way, the average inventory change was 60% of the average GNP decline." (p. 85.)

For the interwar recessions, he notes that inventory investment declined for each of the recessions even though real GNP increased during two of these.

Blinder and Maccini (1991), using data through the 1981-82 recession, detect an 87% average contribution of inventory investment to the peak to trough change in real GNP during post-war recessions. They also decompose the monthly inventory stocks and inventory investment by trade sector to determine which sectors contribute most to the volatility. A more detailed discussion of the extent of the relative movements can be found in Blinder (1990) and Blinder and Maccini (1991).

Stylized Facts and Survey

The primary stylized facts that require explanation according to Blinder (1990 Ch. 6), and Blinder and Maccini (1991) are:

1) production varies more than sales;

2) the covariance of sales and inventory investment is usually found to be positive rather than negative;

3) the empirical speed of adjustment of inventory stocks is usually very slow;

4) changes in inventories of finished goods, work in progress and materials and supplies have very low correlation.

This paper restricts attention to the first two stylized facts. The literature has taken two different directions in confronting these stylized facts. One school seeks to salvage the production smoothing hypothesis by explaining the conditions under which production smoothing can give rise to the observed aggregate stylized fact. The other school focuses on an alternative to the production smoothing paradigm, the most eligible being the (S,s) inventory policy. Work by Blinder, Fair, Braun and Krane, Lai, Kashyap and Wilcox, and Kahn take the first tack. Blinder and Maccini, Caplin, Mosser and Ramey[2] look at (S,s) economies as an alternative.

Blinder (1990 ch. 6) offers a model in which these stylized facts can be explained by the existence of cost shocks, the anticipation of demand shocks by producers, demand shocks that build before they decay, or serially correlated demand. By introducing cost shocks into the model, agents are able to take advantage of intertemporal substitution in production. This substitution can result in higher variance of production than demand. If demand has an autoregressive component and/or firms anticipate demand shocks in their production plans, then the variance of production to sales will be close to unity and the presence of relatively small cost shocks can result in a variance ratio greater than 1.0.

Eichenbaum (1989) empirically tests whether firms engage in production-level smoothing or production-cost smoothing and finds more support for production-cost smoothing than for production-level smoothing. Fair (1989) suggests that lack of good data and complications associated with the sampling interval are the major reasons for finding higher variances of production than variances of sales. He finds strong support for production smoothing with respect to future sales for four of seven industries for which physical-units data are available. Braun and Krane (1991) also find support for production smoothing using physical units data in a wider range of industries. They were not successful in identifying quadratic costs however. Froeb and Koyak (1994) suggest an alternative measure for determining relative smoothness in time series using spectral analysis. Using the new measure they retested 34 of 38 series tested by Braun and Krane (1991) and detected smoothness in some series whose variance ratios were greater than 1.0. Kahn (1987) shows that if demand is positively serially correlated or if firms are allowed to backlog excess demand, production smoothers can experience higher variability in production than sales.

The upshot of these results is that there appears to be some circumstances under which production smoothing agents may not appear to be smoothing in the aggregate. In particular, an increased variance ratio may be observed in agents applying smoothing rules but subject to unforecasted shocks. In addition,

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[2] Ramey tests empirically for nonconvexities in production costs which motivate lumpy behavior.

using the variance ratio as a measure of variability may not be valid for aggregated data. While none of the results provide convincing evidence that smoothing is not a viable paradigm at the firm level, they do not provide strong evidence that production smoothing must be the only prevailing paradigm.

Ramey (1991) concentrates on whether firms face declining marginal costs, which would lead them to "bunch" production rather than smooth. She argues that internal labor market characteristics and capital utilization can lead to declining marginal costs at some levels of production. Ramey tests for declining marginal costs in six manufacturing industries identified as producing to stock and the automobile industry by estimating parameters for a quadratic cost function from a stochastic Euler equation. Her empirical work finds declining marginal costs in the form of a negative quadratic coefficient in all seven industries.

Blinder and Maccini (1991) consider microeconomic theories for inventory investment that adequately address the stylized facts. They acknowledge a variety of reasons for holding inventories:

> Inventories can be held for display purposes, as unavoidable 'pipeline' inventories, to improve production scheduling, to smooth production in the face of fluctuating sales, to minimize stockout costs, to speculate on or hedge against price movements, to reduce purchasing costs by buying in quantity, to shorten delivery lags, and so on. It is clear that no single model can hope to explain the rich variety of inventory behavior; an explanation that is plausible for one industry or type of inventory may be implausible for another.

They nevertheless proceed to seek a unifying abstraction for the microeconomic theory of inventory investment. They recommend the (S,s) model because it appears to resolve the stylized facts more effectively than the production smoothing paradigm, and in part because movements in retail inventory and manufacturing materials and supplies account for the majority of the variation in total trade inventory investment. These two components appear to be good candidates for intermittent adjustment because of the potential for economies of scale in order size.

Caplin (1985) provides a general theory of the aggregation of agents who use (S,s) inventory policies, demonstrating that the variance of orders will exceed the variance of sales under these circumstances. Using a Markov process model in continuous time, he concludes that in the long run the inventory levels of individual retailers are mutually independent, regardless of the correlation in sales. The implication is that there can be no induced dependencies of the movement in inventories from correlation in sales. This result allows characterization of the aggregate implications of (S,s) policies.

Caballero and Engle (1991) provide a framework for analyzing the aggregate dynamics of (S,s) economies. In particular, Caballero and Engle state conditions under which (S,s) economies achieve a steady state, where steady state is defined as a condition in which the distribution of inventories are invariant to the distribution of demand. One of these conditions states that if the agents' inventory are initially uniformly distributed in the (S,s) interval, and if sales are subject to aggregate (correlated) shocks, the inventories will remain uniformly distributed on the (S,s) interval. This coincides with Caplin's observation regarding independence.

Mosser (1991) uses the results of Caplin as a base to empirically test whether aggregate retail data reflect (S,s) decision rules. Her results show consistency with (S,s) but not with quadratic cost models. The underpinnings of the empirical work are based on the continuous time result of independence of individual retailer's level of inventory.

Kahn's (1987) stock-out avoidance model blends the idea of production smoothing with the minimum stock (s) portion of the (S,s) model and explains the higher volatility of production this way. The presence of a penalty for stocking out suggests some minimum stock level, and adjusting to this level would tend to result in the type of variation implicit in (S,s) models. This has some intuitive appeal if there is a cost curve which is convex but has a large penalty for stock-outs.

Further research is necessary to develop the theoretical base for a general solution to this problem. The complexity introduced by the ability to substitute production intertemporally suggests that a general analytic solution is unlikely.

An alternative to choosing between two relatively polar options is to recognize the potential heterogeneity of firms as a result of varying technology, asymmetric information or other factors. Caplin (1985) and Blinder and Maccini (1991) acknowledge that using a multi-sector approach may be useful and recommend it for further research. Several studies examine multi-sector models involving inventory, but most are input-output type models rather than "transactional" models. That is, although production of inputs in one sector for use in another sector is modeled to reflect the appropriate time delays, the models do not usually seek to analyze the dynamic impact of this linkage on the flow of goods. Long and Plosser (1987), for example, use a multi-sector input-output model where output of one sector is an input to another sector, with a one period delay serving as the dynamic link between sectors. This model is used to determine the impact of sectoral and aggregate technological shocks on the aggregate economy. The results show that business cycles can be generated from sectoral shocks as opposed to aggregate shocks.

More recently, Cooper and Haltiwanger (1990) use a multi-sector, two-period model, with one sector holding inventories, to contrast the dynamics of a representative agent, Real Business Cycle (RBC) model with imperfect competition. The intertemporal linkage in this model is the demand for final goods. They conclude that an RBC representative agent, perfectly competitive economy shows more substitution of factors and less positive correlation of employment and output fluctuations across sectors than the imperfectly competitive model. This is interpreted as an indication that an imperfectly competitive model is superior to the competitive RBC model.

Lovell (1993) performs a multi-sector simulation involving an input/output system with 21 industries and four firms in each industry to investigate inventory dynamics. Lovell's simulation assumes firms base purchases of materials and supplies on an (S,s) rule. Finished goods are assumed to be smoothed relative to sales; final demand is determined by a marginal propensity to consume and exogenous government purchases. His simulations demonstrate the potential for complex behavior in the aggregate. Lovell concludes that the ratio of the variance of output to the variance of sales is not a good measure of the amount of smoothing taking place. He also concludes that the flexible accelerator model and the single-firm representation of a multi-sector are reasonable models. Lovell's simulation is of a very complex system, and it is not clear at times which properties of the system are driving the behavior.

The motivations for holding inventory are so diverse that it does not seem plausible that there is one overriding paradigm that can be assumed at the micro level. The class of problems involving optimal inventory levels is large and generates a significant literature by itself. The concept that I wish to bring to the discussion is that built-in buffers and time delays in the vertical hierarchy would distort (amplify or dampen) comovements in aggregate inventory investment, sales and production.

The multi-sector model emphasizes the impact of the vertical linkage on the accumulation and decumulation of inventories and demonstrates the resulting effect on the aggregate movement. I use four identical representative agents at each of three levels to limit the complexity of the simulation sufficiently that the effect of the coupling can be isolated. The agents are assumed to be susceptible to either aggregate or idiosyncratic demand shocks or a combination of both as a way to isolate some of the impact of horizontal aggregation. Unlike Lovell's model, which is a closed model, I assume demand is random and exogenously determined. I assume a variety of inventory rules to show that the stylized facts do not preclude the existence of production smoothing. I concentrate on explaining the first two facts: that production varies more than sales and that inventory investment and sales are positively correlated. The paper will use simulations to support the two hypotheses that these two stylized facts can be explained in part by: 1) the existence of heterogeneity in the inventory decision process and 2) the vertical interaction of agents.

3. MODEL

The model presented here reflects the heterogeneity of firms; it incorporates the differences in the decision rules used to manage inventory or production. I assume the market is made up of three kinds of firms:

1. Manufacturers who sell to wholesalers and retailers.[3]
2. Wholesalers who buy from manufacturers and sell to retailers.
3. Retailers who buy from wholesalers and sell to final consumers.

Manufacturers are assumed to base adjustments to production on demand from wholesalers and on a vector of other factors. Wholesalers are assumed to base their purchases from manufacturers on their

[3] Manufacturers are assumed to be vertically integrated so that purchases of materials and supplies and work in progress is not treated here. A firm with different stages of production would be a micro version of a vertical heterogeneous model where different levels of one firm interact.

sales to retailers as well as a vector of other factors. Finally, retailers are assumed to adjust their purchases from wholesalers based on their final consumer demand and a vector of other factors. In each case the vector of other factors reflects the fact that inventory investment decisions are part of a nexus of profit-seeking decisions.[4]

In determining an appropriate level of inventory, a firm considers the costs of holding inventory, future level of sales, stock-out costs, fixed costs of ordering, delivery times and production costs. In addition, the financial status of the firm must also be considered; liquidity and credit constraints may directly affect the choice of inventory level. For purposes of this paper I restrict my attention to the response due to changes in demand to demonstrate the impact of coupling.

The production/purchasing decision rules of the firms can be represented as:

$$\begin{aligned} Q_1(t) &= f(x_1(t), \overline{z_1}(t)) \\ Q_2(t) &= g(x_2(t), \overline{z_2}(t)) \\ Q_3(t) &= h(x_3(t), \overline{z_3}(t)) \end{aligned} \tag{1}$$

where Q_1 represents production, x_1 manufacturing sales, Q_2 $(= x_1)$ wholesale purchases, x_2 wholesale sales, Q_3 $(= x_2)$ retail purchases, x_3 retail sales, and $\overline{z}_1$, $\overline{z}_2$ and $\overline{z}_3$ are other factors affecting the production and purchasing decisions.

By the accounting rule for inventory:

$$\begin{aligned} \Delta I_1(t) &= Q_1(t) - x_1(t) = f(x_1(t), \overline{z_1}(t)) - x_1(t) \\ \Delta I_2(t) &= Q_2(t) - x_2(t) = g(x_2(t), \overline{z_2}(t)) - x_2(t) \\ \Delta I_3(t) &= Q_3(t) - x_3(t) = h(x_3(t), \overline{z_3}(t)) - x_3(t) \end{aligned} \tag{2}$$

From the dynamic interaction between retail sales and all the other sales and purchases we see that the above inventory equations can be expressed as:

$$\begin{aligned} \Delta I_1(t) &= F(x_3(t), \overline{z_1}(t), \overline{z_2}(t), \overline{z_3}(t)) \\ \Delta I_2(t) &= G(x_3(t), \overline{z_2}(t), \overline{z_3}(t)) \\ \Delta I_3(t) &= H(x_3(t), \overline{z_3}(t)) \end{aligned} \tag{3}$$

[4] Hay and Louri (1991) propose that most inventory analysis tends to overlook this fact and incorporate it in their empirical analysis. They treat inventory investment as part of a portfolio of assets and liabilities which are adjusted based on expected returns. They test their model using UK data and find it to be superior to the traditional stock adjustment model. Work by Carpenter, Fazzari and Petersen (1994) focuses on the substitutability of inventory investment for external financing in imperfect financial markets. They find evidence that firms that are financially constrained may reduce inventory investment more significantly during recessionary periods when internal financing alternatives become scarcer. In spite of these considerations, the focus of the simulations here is restricted to the portion of the decision affected by sales.

where the complexity of the new functions F, G, and H depends on the decision rules. The implicit assumption is that retailers only purchase from wholesalers. It should be noted that the problem statement allows for a variety of decision rules - stock adjustment, (S,s) or ad hoc. Given the above relationships, aggregate inventory change is the summation of F(.), G(.) and H(.).

The result of this interdependence is that even when the dynamic response of production to a change in demand may be dampened in a given sector, the dynamic coupling of these sectors can result in an amplified aggregate dynamic response of production. Thus, although production smoothing may be taking place at the manufacturing level, the aggregate response can be distorted in the transient dynamics due to coupling with the retail or wholesale level, where bunching may be occurring.

Example

For illustrative purposes, consider the impact of a change in retail sales on total inventory levels when the decision rules are second order ARMA's. That is, purchases and production are based on the changes in inventory over the last two periods. For simplicity we will eliminate the wholesale level.

$$Q_i(t)=Q_i(t-1)-\delta_i[Q_i(t-1)-x_i(t-1)]-\rho_i[Q_i(t-2)-x_i(t-2)] \tag{4}$$

Using appropriate time series methods we can establish the relationship:

$$\begin{aligned} Q_i &= \left(\frac{\phi_i(L)}{\theta_i(L)}\right)\times x_i \\ Q_i-x_i &= \left(\frac{\phi_i(L)-\theta_i(L)}{\theta_i(L)}\right)\times x_i \end{aligned} \tag{5}$$

where the following polynomials in lag operators are substituted for brevity:

$$\begin{aligned} \theta_i(L) &= 1-(1+\delta_i)L+\rho_i L^2 \\ \phi_i(L) &= \delta_i L+\rho_i L^2 \end{aligned} \tag{6}$$

The first part of equation 5 is a transfer function representation of the relationship between production and sales. The second equation in 5 is the equivalent form for inventory investment. This leads to the following representation for each inventory change:

$$\Delta INV_3=\left(\frac{\phi_3(L)-\theta_3(L)}{\theta_3(L)}\right)\times x_3 \tag{7}$$

$$\begin{aligned} \Delta INV_1 &= \left(\frac{\phi_1(L)-\theta_1(L)}{\theta_1(L)}\right)\times x_1 \\ &= \left(\frac{\phi_1(L)-\theta_1(L)}{\theta_1(L)}\times\frac{\phi_3(L)}{\theta_3(L)}\right)\times x_3 \end{aligned} \tag{8}$$

The total change in inventory is the sum of the two inventory changes and can be expressed as a function of retail sales. This becomes an ARMA(4,4) even though the original individual decision rules of each firm level was an ARMA(2,2).

Intuitively, we can think of an initial shock which resulted in a stationary response over time at the first level. The second level sees not one shock but several as the responses of the first level becomes the shocks of the second level. The response at the second level is now a higher order than the first. Similarly the third level sees shocks which are the output of the second level. Depending on the speed of decay at each level, significant amplification can occur by the third level.

The upshot of this result is that even when the dynamic response of a given level of production is well behaved, the dynamic coupling of these levels can result in poorly behaved dynamic response. Thus although production smoothing may be taking place at each type of firm, the aggregate response can be distorted in the transient dynamics from the vertical coupling of agents.

Figures 3 and 4 illustrate the effect of coupling on the aggregate response of two agents whose decision rules are:

$$x_{1t}=1.5x_{1t-1}-0.707x_{1t-2}+\epsilon_t$$

$$x_{2t}=0.25x_{2t-1}-0.707_{2t-2}+\mu_t$$

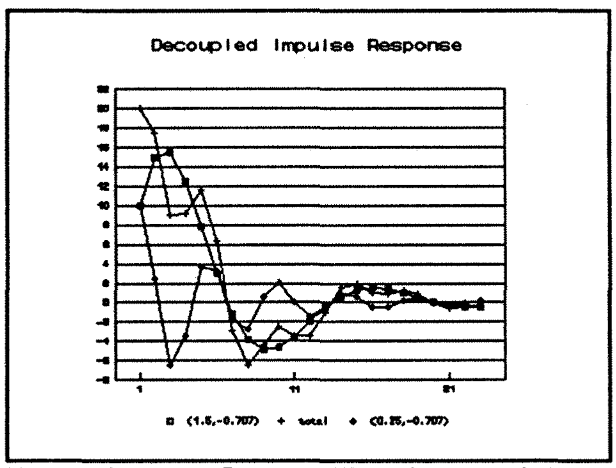

Figure 3 Aggregate Response When Agent1 and Agent2 Unlinked

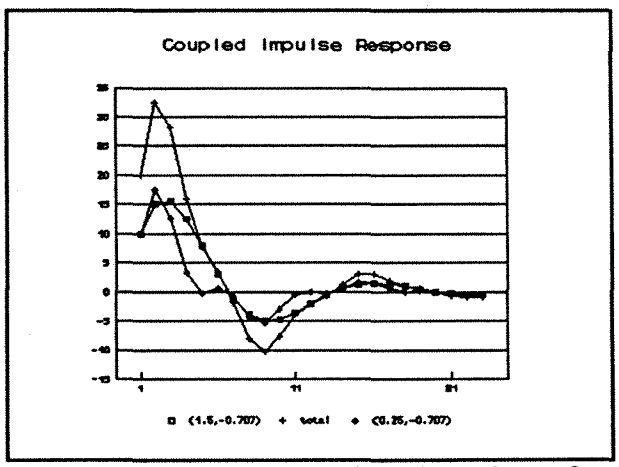

Figure 4 Aggregate Response to Shock When Agent1 Sees Demand From Agent2's Production

Figure 3 shows the summed response when each agent faces the impulse directly. Figure 4 shows the response when agent 2 sees the response of agent 1 as its shock ($\mu_t=x_{1t}$). The two decision rules were chosen to reflect harmonic frequencies where agent 2's response was twice the frequency of agent 1. When they are vertically linked, the amplitude of the aggregate is about 1.5 times as much as when they are unlinked. Figures 5 and 6 compare the aggregate response when both agents have the same decision rule (Agent1's), and are independent, with when Agent 2 responds to demand from Agent 1. In this case the amplitude of the oscillation increases from a peak of about 30 to one of about 85 when agent 2's input is agent 1's response to the shock.

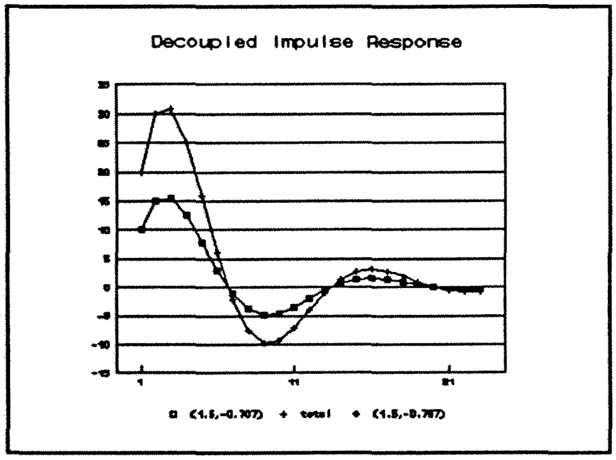

Figure 5 Aggregate Response to a Shock When Agents are Separate but Identical

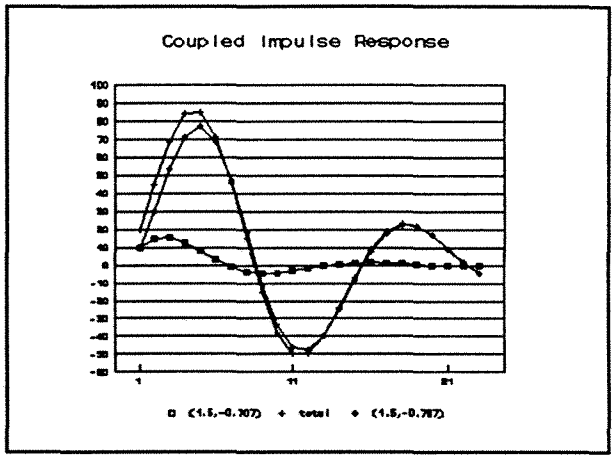

Figure 6 Aggregate Response to Shock When Agent2 Sees Agent1's Response as Demand

4. TEMPORAL AND HORIZONTAL AGGREGATION

The vertical coupling discussed above can lead to dynamic distortion when looking at aggregate data. Typical summary statistics and correlations may be misleading. Similar distortions can also occur when variables are aggregated horizontally and over time. The following authors provide expositions of the theoretical underpinnings for the effects of horizontal and temporal aggregation.

Christiano and Eichenbaum (1987) discuss the implications of temporal aggregation on macroeconomic variables. When the decision period for agents does not coincide with the data sampling intervals, distortions and erroneous conclusions may occur. This bias due to temporal aggregation can lead to rejection of models using conventional statistical tests. They demonstrate how this lack of coordination between sampling interval and decision interval can result in the observed stylized result of slow speeds of adjustment.

Analyzing the impact of horizontal aggregation across (S,s) agents has been shown to be very difficult (Blinder [1990]). Caplin (1985) uses a model of (S,s) which implies mutual independence of inventory levels regardless of the correlation between sales. This result allows him to make statements regarding the long-run aggregate variance. The primary aggregation result is that the difference between the variance of aggregate sales and the variance of aggregate production is the sum of the individual differences of the variances of each agent. The correlation between sales does not play a part. As indicated above, this result hinges on the fact that the continuous time model allows agents to replenish at exactly s. Caballero and Engle (1991) arrive at similar conclusions, but they incorporate a heterogeneity of agents into the (S,s) economies and derive a result for cases where shocks are idiosyncratic or systemic. They also conclude that the economies will approach the steady state in the aggregate independently of the level of idiosyncratic shocks. It appears that the assumption of continuous time and replenishment exactly at s is necessary to get these results because the simulations show a tendency to synchronization when aggregate shocks dominate.

Horizontal aggregation of agents who use the production smoothing rule presents a different challenge. Intuitively, one would expect the aggregation of agents who smooth to result in variance ratios even less than for individual agents. Lai demonstrates, however, that when there are both aggregate and idiosyncratic shocks to sales, the aggregate variance ratio can be distorted, resulting in a higher variance ratio in the aggregate when idiosyncratic shocks dominate. The simulation results are accumulated over time and across agents to test these theoretical results on aggregation.

5. SIMULATION RESULTS

Introduction

The primary objective of the simulations is to illuminate the effect of heterogeneity in the decision rules and vertical interaction of agents on the two stylized facts - that production varies more than sales and that inventory investment and sales are positively correlated. The secondary objectives were to determine the effect of horizontal and temporal aggregation and the relative magnitudes of aggregate and idiosyncratic shocks on these two stylized facts.

Heterogeneity is simulated by allowing for different combinations of two decision rules at each of the three levels of interaction. Five different combinations of decision rules were simulated. The vertical interaction is simulated by the identification of three types of agents. Retailers observe and respond to retail demand; wholesalers observe and respond to retail purchases; and manufacturers observe and respond to wholesale orders. For each set of decision rules, three kinds of demand shocks were then simulated at the retail level - exclusively aggregate shocks, an equal combination of aggregate and idiosyncratic shocks, and finally, exclusively idiosyncratic shocks. Aggregate or systemic shocks are defined as a disturbances which affect each agent equally and simulated as a random draw, v_t, applied to each agents demand; idiosyncratic shocks are defined in this context as independent disturbances (from the same distribution) affecting each agent and are simulated as different draws, v_{it}, from the same distribution applied to each agent's demand.[5] Aggregate and idiosyncratic shocks at the retail level are transmitted to the wholesale and manufacturing

[5] The definition of idiosyncratic shocks used here is slightly different from the case where idiosyncratic shocks are considered "zero-sum" redistributions of demand.

levels by a variation in the amount of sales that reflect an average of the downstream agents versus sales from an exclusive downstream agent.

Table 1 lists the cases that were simulated. Each set of decision rules represent one group of cases. The impact of the different combination of shocks is then isolated for each group. Cases 1, 2 and 3 are classified as Group A, Cases 4, 5 and 6 as Group B, Cases 7, 8, and 9 as Group C, Cases 10, 11, and 12 as Group D, Cases 13, 14, and 15 as Group E.

Table 1 Description of Cases

Group	Case	Retail Decision Rule	Wholesale Decision Rule	Manufacturers Decision Rule	Retail Demand Shocks[6]
A	Case 1	S,s	S,s	Production Smoothing	A
	Case 2	S,s	S,s	Production Smoothing	A/I
	Case 3	S,s	S,s	Production Smoothing	I
B	Case 4	Order Smoothing	Order Smoothing	Production Smoothing	A
	Case 5	Order Smoothing	Order Smoothing	Production Smoothing	A/I
	Case 6	Order Smoothing	Order Smoothing	Production Smoothing	I
C	Case 7	Order Smoothing	Order Smoothing	S.s	A
	Case 8	Order Smoothing	Order Smoothing	S,s	A/I
	Case 9	Order Smoothing	Order Smoothing	S,s	I
D	Case 10	Order Smoothing	S,s	Production Smoothing	A
	Case 11	Order Smoothing	S,s	Production Smoothing	A/I
	Case 12	Order Smoothing	S,s	Production Smoothing	I
E	Case 13	S,s	Order Smoothing	S,s	A
	Case 14	S,s	Order Smoothing	S,s	A/I
	Case 15	S,s	Order Smoothing	S,s	I

The first three cases, Group A, represent the most plausible combination of decision rules in that bunching or S,s behavior is more likely at the wholesale and retail level while production smoothing makes sense at the manufacturing level when quadratic or convex costs obtain. The other sets of decision rules (Groups B through E) reflect less plausible rules for the agents as identified. That is, it may be unlikely that a retailer, faced with high costs of delivery would choose to smooth. However, in the abstract, the simulations can in fact reflect the vertical interaction within a single firm where either smoothing or (S,s) may be the appropriate decision at the division level.

Assumptions

The assumptions made herein do not reflect an attempt to perform a calibration exercise, because my intention is to demonstrate the mechanics of the interaction. The chosen parameters do reflect reasonable parameters and a sensitivity analysis test of the robustness of the conclusions over a range of parameters was performed.

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[6] A=Aggregate Shocks; A/I=Both Aggregate and Idiosyncratic; I=Idiosyncratic

The following assumptions were made for all base case simulations:

o Initial inventory levels were drawn from a uniform distribution between 200 and 500.

o Aggregate and idiosyncratic sales shocks were drawn from a normal distribution with mean zero and variance 225.

o Retail sales were assumed to be 100 plus the appropriate combination of aggregate and idiosyncratic shocks.

o (S,s) inventory rules specified that if inventory levels fall below 200, purchases (or production) were made to return inventory to 500 at the beginning of the next period.

o The production smoothing rule was assumed to be a three period moving average of past sales. There is no attempt to tie this to optimizing behavior. Intuitively, however, given a demand pattern which reflects customers who replenish every three periods, production smoothed to a three-period moving average would reflect at least a near optimal smoothing.

o 100% of retail purchases were made from wholesalers, and 100% of all wholesale purchases were made from manufacturers.

o Wholesale and manufacturers' sales were assumed to be a combination of the average purchases of all 4 "downstream" agents and the purchase of the immediate "downstream" agent. That is, the i^{th} wholesale agent sees sales of either 0.25 times the total retail purchases (i.e. the average) in the cases of aggregate shocks, 100% of the i^{th} retail agents purchases in the case of purely idiosyncratic retail demand shocks, or a combination of the two when a combination of shocks is imposed at the retail level. This was done instead of introducing additional idiosyncratic shocks at the wholesale and manufacturing level. In all cases total wholesale sales equal total retail purchases and total manufacturing sales are equal to total wholesale purchases.

The interaction of 4 identical firms at each level was simulated for 100 periods. Although the periods can be considered to be months, they are not necessarily intended to be. The resulting means and variances of the sales, production or purchases, and inventory levels were recorded for first differences, aggregated over agents in each sector and then temporally aggregated over three periods (which can be considered to be a quarter).

Results

Cases 1, 4, 7, 10 and 13 have exclusively aggregate shocks to retail sales. Aggregate shocks to retail sales increase the likelihood of synchronization of the purchases by agents who use an (S,s) rule. After a number of periods, the aggregate of the four firms behaves as one firm using the (S,s) rule.[7] This

[7] It was observed in a separate experiment in horizontal aggregation of 100 (S,s) agents that large aggregate shocks again resulted in synchronized movement after a number of periods.

occurs because the firms do not replenish exactly at s but at some s - ϵ_{it}, with the result that, depending on the size of the shock, more than one firm can end up replenishing at the same time. Once they replenish together once, they will synchronize from then on, because each firm replenishes to the same S. Over a long enough period of aggregate shocks, the individual firms eventually all synchronize and move as one composite agent. The resulting variances of (S,s) agents are highest with aggregate demand shocks. This can be observed by noting the height of the sawtooth for aggregate (S,s) purchases in the figures in the appendix. When purchases during one period exceed the 1200 composite (S-s) for four firms, this is an indication that their inventory had dropped substantially below s in the period before.

As idiosyncratic shocks are added, the variance of the retail sales is lowered, and there is also less synchronization. The variance of (S,s) firms purchases is lowered by the diversity of replenishment times. In Cases 2, 5, 8, 11 and 14, the variance of the retail demand decreases, reflecting a combination of aggregate and idiosyncratic shocks. The impact on the variance ratio depends on the relative reduction in the variance of the firms' production or purchases. In most cases, the reduction in the variance in sales was more than the reduction in the variance of retail purchases resulting in an increase in the variance ratio. In Cases 3, 6, 9, 12 and 15 retail demand is subjected to idiosyncratic shocks only. This reflects the most diversity between the purchases of the (S,s) agents. Here again, the net effect on the variance ratio is indeterminate.

The smoothing rule used throughout matched closely the (S,s) purchasing patterns. (S,s) agents replenished every 3 periods on average, and Production/order smoothing agents smoothed over three periods. The ratio of the variance of production to the variance of sales whenever smoothers face demand from (S,s) agents is much less than one. However, in Group A for example, even though the ratio of the variance of manufacturing purchases to the variance of manufacturing sales is much less than 1.0, the large variances induced by the use of (S,s) rules at the retail and wholesale levels result in a ratio of the variance of manufacturing production to <u>retail</u> sales that is much greater than one. This demonstrates that the presence of downstream S,s agents can mask the existence of smoothing when aggregate data are observed.

Cases 4, 5 and 6 represent a homogeneous set of agents who all use a smoothing rule but are connected vertically. Typically one might expect to find this arrangement when manufacturers produce inputs for other manufacturers. This arrangement is less likely with actual retailers and wholesalers. It serves to isolate the impact of the vertical interaction as distinguished from heterogeneity.

Summary Statistics

The summary statistics (means, variances and variance ratios) were compiled for the aggregate of each level. For a given combination of decision rules we observe the variance ratios and correlations of sales to inventory at three different levels of aggregation to isolate the effect of aggregation. Summary statistics were compiled for individual first differences, aggregated over individuals and then temporally aggregated over three periods.

Table 2 summarizes the means and variances of the aggregate of each level as well as totals for Cases 1, 2 and 3. The variances of production or purchases of firms using (S,s) rules are primarily a function of the level of sales and the time to replenishment. This is reflected in the variance ratio of retail purchases to retail sales ($\sigma_{rp}^2/\sigma_{rs}^2$) being very large.[8] The ratio of the variance of retail purchases to sales ranges from 81.0 when only aggregate shocks are present to 91.5 when only idiosyncratic shocks are present. This reflects both the impact of the swings in purchases resulting from the intermittent adjustment and the reduction in variance of retail sales with the increased diversity. When idiosyncratic shocks become more prominent, there is a canceling effect and the variance of retail sales decreases. The variance of retail purchases and wholesale purchases are also reduced as idiosyncratic shocks increase, because there is less synchronization of agents' purchases.

The wholesalers also purchase using an (S,s) rule, but the variance of wholesale purchases relative to the retail purchases ($\sigma_{wp}^2/\sigma_{rp}^2$) range between 1.1 and 2.23. This result emphasizes that the variance is dominated by the mean sales and time to replenishment since the mean sales and time to replenishment are approximately the same for both levels.

In all three cases, the variance ratio of manufacturers production relative to retail sales ($\sigma_{mp}^2/\sigma_{rs}^2$) ranges from 9.4 and 20.6, even though the manufacturers smooth production relative to wholesale sales. This illustrates that the presence of downstream lumpy adjustment masks production smoothing when compared to final sales. Figures A1-A3 in the appendix show the dynamic movement of the variables for Cases 1-3 respectively.

Cases 4, 5 and 6 (Group B) reflect decision rules where all 3 types of agents use a purchase or production smoothing rule. The summary statistics are shown in Table 3. In Case 4 the demand shocks are aggregate shocks; in Case 5 they are a mixture of aggregate and idiosyncratic shocks; and in Case 6 they are purely idiosyncratic. The smoothing rule in place is the same as that used by manufacturers in Cases 1, 2 and 3. The rule is a three period moving average of sales, which worked quite well with lumpy sales from a wholesale purchase rule that reflected an average time to replenishment of three periods. Now, however, the retailers are using this smoothing rule in the face of sales that are Gaussian disturbances around a mean and the smoothing rule is not as effective in this case. The first noticeable result is that the variance ratio of retail purchases to sales is greater than one. This implies immediately that a smoothing rule which is optimal (or at least near optimal) for a given set of disturbances may not be appropriate when

[8] In the aggregate economy, variances would tend to be less due to aggregation over many more agents.

some other disturbance occurs. The second noticeable result is that the "downstream" variance ratios are approximately 1.0. This could be because the smoothing rule also reflects a 100% adjustment over three periods. The third point of interest in these results is that the variance ratio of the retail level increases as shocks become more idiosyncratic. This verifies Lai's aggregation bias results and Lovell's conclusions that the variance ratio may not be a reliable measure of the degree of smoothing taking place at the firm level. Figures A4-A6 show the dynamic movement of the variables over the period.

Table 4 shows the results for Cases 7, 8 and 9 (Group C) where the decision rules at the wholesale and retail levels are to smooth purchases and the manufacturing level uses an (S,s) production rule. The demand shocks are aggregate, a mixture of aggregate and idiosyncratic and purely idiosyncratic for Cases 7, 8 and 9 respectively. The primary interesting result here is the reduction in the variance of production to sales when the shocks are more idiosyncratic. As expected the variance is largest for the (S,s) rule. Also notable is the fact that with manufacturing using an (S,s) rule, the overall variance ratio exceeds Cases 1 through 6. The dynamics of Group C variables are shown in Tables A7-A9 in the appendix.

Table 5 shows the summary statistics for Cases 10, 11 and 12 (Group D) where the decision rules are smoothing at the retail and manufacturing level and (S,s) adjustment at the wholesale level. Here again the decision rule at the wholesale level matches the smoothing rule at the manufacturing level, resulting in a variance ratio at the manufacturing level of 0.1, which shows effective smoothing. The same rule at the retail level, which faces a Gaussian disturbance, results in greater variance ratios. Despite the existence of two levels of smoothing, however, the variance ratio of manufacturing production to retail sales remains significantly greater than one. This supports the conclusion that the existence of one level which uses the (S,s) rule is sufficient to result in a variance ratio resembling (S,s) behavior. In this instance, however, because smoothing is occurring before and after the (S,s) level, the overall variance ratio is not as high as in Cases 7-9. Figures A10-A12 show the sales, production and inventory movements over the period.

Finally, Cases 13, 14 and 15 (Group E) simulate a scenario where manufacturers and retailers use an (S,s) rule while wholesalers use a purchase smoothing rule. Again in this scenario the smoothing rule at the wholesale level is matched to the characteristic of the downstream purchasing pattern and results in a variance ratio of 0.1. As in Cases 7,8, and 9, the use of an (S,s) rule at the manufacturing end again results in a higher overall variance ratio than the case where smoothing is taking place at the manufacturing level. Table 6 shows the summary statistics and Figures A13-A15 show the dynamic movement of the variables in this group.

Aggregation Effects

Multiple agents at each level were assumed to identify possible aggregation effects. Some authors have suggested that there is no aggregation bias in the (S,s) situation under certain conditions. This is counterintuitive because one would expect the presence of many agents who use the (S,s) rule and replenish at random times to have a canceling effect on the variation of the aggregate. Similarly, as one aggregates over time it seems intuitive that the sawtooth pattern of lumpiness would be smoothed out. On the other hand, it is not clear intuitively how aggregation affects the production smoothing rule. One might expect that the aggregation of data from agents who smooth would result in a higher degree of smoothing in the aggregate data. Lai shows that the aggregation bias in production smoothing is a function of the relative size and variance of aggregate and idiosyncratic shocks. More specifically, when idiosyncratic shocks that affect the relative market share of each firm are larger than common shocks, the variance in aggregate production increases relative to the variance in aggregate sales when firms partially adjust production in an effort to smooth. Lai concludes that aggregation bias will be more important in cases when aggregate shocks are less significant.

The statistics for each scenario simulated were accumulated at different levels of aggregation to detect any general trends in the impact of aggregation. The overall results support the assertion that aggregation bias is important in testing the degree of smoothness. The results indicated a downward aggregation bias in the variance ratios of (S,s) agents and an upward bias in the aggregation of production smoothing agents. At each level of the economy and regardless of the type of shocks, the ratio of the variance of production to the variance of sales decreased as the variables were aggregated over agents and temporally when agents used an (S,s) rule. Similarly, at all levels, when the variables were aggregated for production smoothers, the variance ratio increased. This result verifies the theoretical results of Lai and supports the simulation results of Lovell, which again suggests that a ratio of the variances of production to the variance of sales may be a poor measure of the degree of smoothing.

At the individual agent level, the variance of first differences were accumulated. The statistics reported in the first section of Tables 2 through 6 are for the sum of all four agents at each level. These ratios are repeated and accumulated over 3 periods and shown as "quarterly", (i.e. implicitly assuming each period is equivalent to one month.) The summary statistics of the first differences showed less of the effects of aggregation.

Inventory Investment Correlation with Sales

The correlation coefficients of levels and first differences of sales and inventory were recorded to see how the second stylized fact held for the simulated scenarios. For the most part the expected negative correlation between changes in sales and changes in inventory were observed, verifying the buffer-stock nature of inventory. That is, in most cases as sales increased, inventory decreased. In some instances the correlation was positive. However, there were no obvious patterns discernible from the data.

In the simulation cases where the agent used production smoothing and their clients also used production smoothing, the correlation turned out to be positive. Temporal aggregation (over three periods) tended to increase the correlation coefficient for (S,s) agents. In cases where the negative correlation of sales to inventory for the (S,s) agents was relatively small, temporal aggregation tended to change to positive correlation. As in the case of the first stylized fact, a caution flag must also be waved when observing the correlation of aggregate data. Aggregate data which includes (S,s) agents tend to be more positively correlated than disaggregated data. In this case, however, it would appear that aggregation would not normally bias the sales/inventory correlation for production smoothers from the predicted negative, except when there is vertical interaction with other smoothers. In other words, if the demand is from customers who smooth by partially adjusting, then there is increased likelihood that there will be bias toward positive correlation in the aggregate.

For cases 1, 2 and 3, the retail sales to inventory showed low negative correlation for the aggregate of all retail agents. When aggregated over 3 periods the correlation became positive, indicating some bias from aggregation. Temporal aggregation biased the correlation towards positive correlation for (S,s) agents whereas for the most part it biased the correlation for smoothers more negatively. In other words, quarterly sales to inventory correlation coefficients were more negative than monthly for production smoothers and less negative than monthly for (S,s) agents.

Positive correlation between sales and inventory were observed for:

o Quarterly retail using (S,s) decision rules with aggregate shocks,

o Monthly wholesale with idiosyncratic shocks and the downstream retailers also using production smoothing,

o Monthly manufacturing using production smoothing with aggregate shocks and all downstream wholesalers and retailers also smoothing.

Tentative conclusions that can be made are that when downstream agents (i.e. customers) use smoothing rules there is a tendency for increased positive correlation of sales and inventory; (S,s) decision rules also tend to increase the positive correlation of sales and inventory; and aggregation tends to increase the positive correlation for (S,s) agents and increase the negative correlation for smoothing agents.

Summary and Conclusions

The results of the simulations show the potential for unmodeled dynamics in the economy to distort the statistics and lead to erroneous inference. In particular, the simulations showed that:

o when some agents use (S,s) inventory decision rules, the induced variance ratios will dominate the aggregate data.

o when data are aggregated the impact is to lower the variance ratio for (S,s) agents and to increase the variance ratio for production smoothers.

o sales and inventory were negatively correlated for the most part, but tended toward positive correlation when downstream agents or customers were also smoothing.

The paper set out to address two stylized facts concerning inventory: production varies more than sales and the correlation between sales and inventory is positive instead of negative. The results of these simulations contribute the following explanations for these stylized facts:

o The existence of a level within the vertical hierarchy that uses a lumpy adjustment rule is sufficient to result in the variance of the aggregate output being greater than the variance of aggregate demand.

o Bias in aggregation tends to increase the variance ratio of production smoothing firms and decrease the variance ratio of agents using (S,s) decision rules.

o Bias in aggregation reduces the negative correlation between sales and inventory for (S,s) agents and increases the negative correlation for production smoothing agents.

o When aggregate shocks dominate, agents who use (S,s) rules tend to synchronize, which contributes to the increased variability of purchases compared to sales.

o The simulations showed negative correlation between sales and inventory investment in general. However in the cases where production smoothers faced demand from other production smoothers there tended to be positive correlation between sales and inventory investment.

These findings suggest that the production smoothing alternative should not be abandoned but that a disaggregated model reflecting heterogeneous rules might be more appropriate to model inventory investment dynamics. In particular, a few researchers have questioned whether it is valid to use the ratio of the variance of production to the variance of sales as an indicator of the level of production smoothing taking place. It would appear that there are several situations which would lead to a variance ratio that exceeds 1.0. The aggregation bias would suggest that tests of production smoothing with aggregate data using the variance ratio as a measure should be done with caution. There may in fact be no contradiction between the observed higher volatility in production than sales at the aggregate level with the microeconomic motivation of smoothing.

Since the observed variance ratios for disaggregated industries are mixed, it is conceivable that the differences reflect either heterogeneity or aggregation bias. Detailed observation of the production cost characteristics of individual firms in conjunction with observation of the variance ratios of these firms might provide insight into the effects of heterogeneity.

What if anything does this suggest about the variability of inventory investment during contractions? First the potential for synchronization of (S,s) agents when there is a dominance of aggregate shocks suggests that contractions may reflect a simultaneous delaying of orders by agents who use (S,s) inventory decision rules. In addition the observed rapid growth after the downturn may reflect synchronization of replenishment by (S,s) agents. Secondly, the vertical interaction of agents may introduce a transient dynamic response of production and inventory to an impulse shock in sales which is exaggerated in the aggregate. Finally, despite the limitations inherent in making generalized conclusions from simulations, it would appear that modeling heterogeneity is important to understanding the dynamics of inventory investment.

Table 2 Group A Means and Variances of Production, Purchases, Sales and Inventory Production Smoothing at the Manufacturing Level, (S,s) Rules at the Wholesale and Retail Levels

	Retail Sales	Retail Purchases	$\sigma_{rp}^2/\sigma_{rs}^2$	Wholesale Purchases	$\sigma_{wp}^2/\sigma_{rp}^2$	Manufact Production	$\sigma_{mp}^2/\sigma_{wp}^2$	$\sigma_{mp}^2/\sigma_{rs}^2$	Retail Inventory	Wholesale Inventory	Manufacturing Inventory	Total Inventory
Case 1												
Mean	401.0	387.1		397.1		416.9			1096.1	1464.3	2384.9	4945.4
Variance	3699.6	299660.7	81.0	418607.6	1.4	34704.7	0.08	9.38	118983.7	356028.5	256296.7	44048.4
Case 2												
Mean	400.6	391.9		401.9		412.2			1103.9	1315.8	2389.0	4808.8
Variance	1281.0	112328.8	87.7	250297.8	2.23	26393.3	0.11	20.6	48777.3	196825.3	169157.1	39130.1
Case 3												
Mean	400.2	394.5		394.0		413.2			1112.8	1563.1	2357.3	5033.1
Variance	1280.0	117146.0	91.5	130457.9	1.11	18202.1	0.14	14.2	46816.5	144834.8	99182.3	69226.4

Variance Ratios and Correlation Coefficients

	Variance Ratio				Correlation Coefficient		
	Retail	Wholesale	Manufacturing	Mfg/Ret	Retail	Wholesale	Manufacturing
Case 1							
Avg. Indiv. First Differences	182.0	1.42	0.06				
Aggregate of Agents	81.0	1.40	0.08	9.4	-0.038	-0.918	-0.762
Temporal ("Quarterly") Aggregate	16.9	1.3	0.4	6.8	0.614	-0.205	-0.666
Case 2							
Avg. Indiv. First Differences	289.6	2.38	0.07				
Aggregate of Agents	87.7	2.23	0.11	20.6	-0.187	-0.765	-0.747
Temporal ("Quarterly") Aggregate	45.7	1.77	0.67	30.8	0.405	-0.107	-0.725
Case 3							
Avg. Indiv. First Differences	132.5	1.06	0.06				
Aggregate of Agents	91.5	1.11	0.14	14.2	-0.112	-0.854	-0.723
Temporal ("Quarterly") Aggregate	49.5	1.33	0.74	36.7	-0.188	-0.028	-0.846

Table 3 Group B Means and Variances of Production, Purchases, Sales and Inventory Smoothing at All Levels

	Retail Sales	Retail Purchases	$\sigma_{rp}^2/\sigma_{rs}^2$	Wholesale Purchases	$\sigma_{wp}^2/\sigma_{rp}^2$	Manufact Production	$\sigma_{mp}^2/\sigma_{wp}^2$	$\sigma_{mp}^2/\sigma_{rs}^2$	Retail Inventory	Wholesale Inventory	Manufacturing Inventory	Total Inventory
Case 4												
Mean	401.0	406.4		411.8		416.7			2334.1	3081.6	3678.6	9094.0
Variance	3699.6	6180.0	1.67	6683.9	1.08	7711.1	1.15	2.08	13471.6	70027.9	73284.2	379894.6
Case 5												
Mean	400.6	405.1		410.3		415.4			2342.0	3084.1	3681.4	9107.5
Variance	1281.0	5158.6	4.03	6015.1	1.17	7186.8	1.2	5.61	8943.2	67330.6	71260.1	368233.7
Case 6												
Mean	400.2	403.8		408.8		414.0			2349.8	3087.0	3684.2	9120.9
Variance	1280.0	5128.5	4.0	6053.4	1.18	7258.0	1.2	5.67	9054.2	67613.2	71685.7	373943.9

Variance Ratios and Correlation Coefficients

	Variance Ratio				Correlation Coefficient		
	Retail	Wholesale	Manufacturing	Mfg/Ret	Retail	Wholesale	Manufacturing
Case 4							
Avg. Indiv. First Differences	1.52	0.87	0.99				
Aggregate of Agents	1.67	1.08	1.15	2.08	-0.424	-0.030	-0.104
Temporal ("Quarterly") Aggregate	0.99	1.50	2.14	2.14	-0.920	-0.925	-0.928
Case 5							
Avg. Indiv. First Differences	2.24	0.91	1.00				
Aggregate of Agents	4.0	1.17	1.19	5.6	-0.129	-0.046	-0.058
Temporal ("Quarterly") Aggregate	2.4	2.16	3.62	8.8	-0.879	-0.965	-0.937
Case 6							
Avg. Indiv. First Differences	1.09	0.88	0.99				
Aggregate of Agents	4.01	1.18	1.20	5.7	-0.182	0.042	-0.066
Temporal ("Quarterly") Aggregate	2.72	2.23	3.76	10.6	-0.916	-0.966	-0.939

Table 4 Group C Means and Variances of Production, Purchases, Sales and Inventory Smoothing at the Retail and Wholesale Levels, (S,s) at the Manufacturing Level

	Retail Sales	Retail Purchases	$\sigma_{rp}^2/\sigma_{rs}^2$	Wholesale Purchases	$\sigma_{wp}^2/\sigma_{rp}^2$	Manufact Production	$\sigma_{mp}^2/\sigma_{wp}^2$	$\sigma_{mp}^2/\sigma_{rs}^2$	Retail Inventory	Wholesale Inventory	Manufacturing Inventory	Total Inventory
Case 7												
Mean	401.0	406.4		411.8		419.5			2334.1	3081.6	1094.8	6510.2
Variance	3699.6	6180.0	1.67	6683.9	1.08	371459.9	55.6	100.4	13471.6	70027.9	147645.2	256090.0
Case 8												
Mean	400.6	405.1		410.3		406.0			2342.0	3084.1	1096.4	6522.5
Variance	1281.0	5158.6	4.03	6015.1	1.17	200211.7	33.3	156.3	8943.2	67330.6	79541.9	184746.1
Case 9												
Mean	400.2	403.8		408.8		402.6			2349.8	3087.0	1091.0	6527.8
Variance	1280.0	5128.5	4.0	6053.4	1.18	106155.8	1.2	5.67	9054.2	67613.2	43329.2	147188.5

Variance Ratios and Correlation Coefficients

	Variance Ratio				Correlation Coefficient		
	Retail	Wholesale	Manufacturing	Mfg/Ret	Retail	Wholesale	Manufacturing
Case 7							
Avg. Indiv. First Differences	1.52	0.87	145.5				
Aggregate of Agents	1.67	1.08	55.6	100.4	-0.424	-0.030	-0.167
Temporal ("Quarterly") Aggregate	0.99	1.50	17.6	17.4	-0.920	-0.925	-0.680
Case 8							
Avg. Indiv. First Differences	2.24	0.91	149.7				
Aggregate of Agents	4.0	1.17	33.3	156.3	-0.129	-0.046	-0.224
Temporal ("Quarterly") Aggregate	2.4	2.16	13.0	31.5	-0.879	-0.965	-0.396
Case 9							
Avg. Indiv. First Differences	1.09	0.88	145.4				
Aggregate of Agents	4.01	1.18	17.5	82.9	-0.182	0.042	-0.327
Temporal ("Quarterly") Aggregate	2.72	2.23	7.1	19.4	-0.916	-0.966	-0.452

Table 5 Group D Means and Variances of Production, Purchases, Sales and Inventory Smoothing at the Retail and Manufacturing Levels, (S,s) Rule at the Wholesale Level

	Retail Sales	Retail Purchases	$\sigma_{rp}^2/\sigma_{rs}^2$	Wholesale Purchases	$\sigma_{wp}^2/\sigma_{rp}^2$	Manufact Production	$\sigma_{mp}^2/\sigma_{wp}^2$	$\sigma_{mp}^2/\sigma_{rs}^2$	Retail Inventory	Wholesale Inventory	Manufacturing Inventory	Total Inventory
Case 10												
Mean	401.0	406.4		404.2		415.1			2334.1	1088.9	4139.1	7562.1
Variance	3699.6	6180.0	1.67	411241.6	66.7	28415.7	0.07	7.7	13471.6	158725.4	321505.2	247211.7
Case 11												
Mean	400.6	405.1		403.9		414.4			2342.0	1087.6	4140.6	7570.2
Variance	1281.0	5158.6	4.03	165452.5	32.1	12607.8	0.08	9.8	8943.2	60963.7	175583.5	233789.8
Case 12												
Mean	400.2	403.8		391.6		393.8			2349.8	1131.4	3722.1	7203.3
Variance	1280.0	5128.5	4.0	101022.4	19.7	15494.2	0.15	12.1	9054.2	80043.0	136509.8	155916.8

Variance Ratios and Correlation Coefficients

	Variance Ratio				Correlation Coefficient		
	Retail	Wholesale	Manufacturing	Mfg/Ret	Retail	Wholesale	Manufacturing
Case 10							
Avg. Indiv. First Differences	1.52	126.3	0.05				
Aggregate of Agents	1.67	66.7	0.07	7.7	-0.424	-0.100	-0.610
Temporal ("Quarterly") Aggregate	0.99	18.1	4.22	4.2	-0.920	-0.215	-0.665
Case 11							
Avg. Indiv. First Differences	2.24	135.8	0.05				
Aggregate of Agents	4.0	32.1	0.08	9.8	-0.129	-0.136	-0.437
Temporal ("Quarterly") Aggregate	2.4	9.84	4.51	10.9	-0.879	-0.168	-0.721
Case 12							
Avg. Indiv. First Differences	1.09	120.8	0.07				
Aggregate of Agents	4.01	19.7	0.15	12.1	-0.182	**0.456**	-0.530
Temporal ("Quarterly") Aggregate	2.72	14.4	8.1	22.1	-0.916	**0.675**	-0.777

Table 6 Group E Means and Variances of Production, Purchases, Sales and Inventory (S,s) Decision Rule at the Retail and Manufacturing Levels, Smoothing at the Wholesale Level

	Retail Sales	Retail Purchases	$\sigma_{rp}^2/\sigma_{rs}^2$	Wholesale Purchases	$\sigma_{wp}^2/\sigma_{rp}^2$	Manufact Production	$\sigma_{mp}^2/\sigma_{wp}^2$	$\sigma_{mp}^2/\sigma_{rs}^2$	Retail Inventory	Wholesale Inventory	Manufacturing Inventory	Total Inventory
Case 13												
Mean	401.0	387.1		411.1		398.5			1096.1	1797.5	1154.1	4047.8
Variance	3699.6	299660.7	81.0	23888.1	0.08	123073.3	5.15	33.3	118983.7	185021.0	84221.7	67791.9
Case 14												
Mean	400.6	391.9		409.1		393.6			1103.9	1790.1	1099.0	3993.0
Variance	1281.0	112328.8	87.7	14063.9	0.13	234613.4	16.7	183.2	48777.3	78963.7	100396.9	103551.2
Case 15												
Mean	400.2	394.5		406.8		393.0			1112.8	1793.2	1148.2	4054.2
Variance	1280.0	117146.0	91.5	13391.0	0.11	99675.9	7.44	77.9	46816.5	74616.9	53998.3	41742.6

Variance Ratios and Correlation Coefficients

	Variance Ratio				Correlation Coefficient		
	Retail	Wholesale	Manufacturing	Mfg/Ret	Retail	Wholesale	Manufacturing
Case 13							
Avg. Indiv. First Differences	182.0	0.06	24.9				
Aggregate of Agents	81.0	0.08	5.2	33.3	-0.038	-0.748	-0.736
Temporal ("Quarterly") Aggregate	16.9	0.41	0.41	6.9	**0.614**	-0.739	-0.317
Case 14							
Avg. Indiv. First Differences	289.6	0.07	43.9				
Aggregate of Agents	87.7	0.13	16.7	183.2	-0.187	-0.682	-0.341
Temporal ("Quarterly") Aggregate	45.7	0.50	1.25	57.0	**0.405**	-0.797	**0.008**
Case 15							
Avg. Indiv. First Differences	132.5	0.06	17.6				
Aggregate of Agents	91.5	0.11	7.4	77.9	-0.112	-0.708	-0.440
Temporal ("Quarterly") Aggregate	49.5	0.54	0.43	21.5	-0.188	-0.790	-0.507

Appendix

Figure A1 Sales, Purchases and Inventory for Case 1

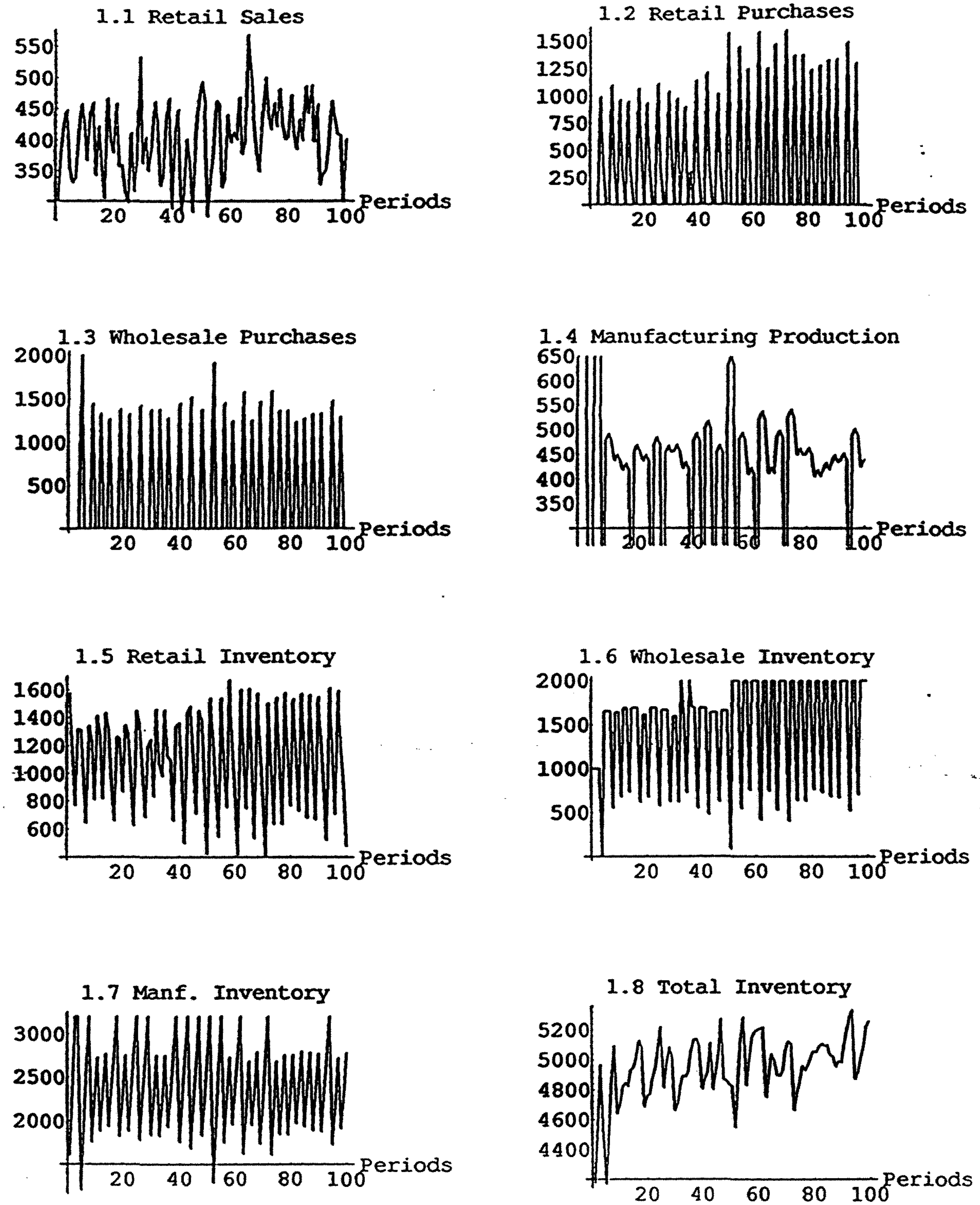


Figure A2 Sales, Purchases and Inventory for Case 2

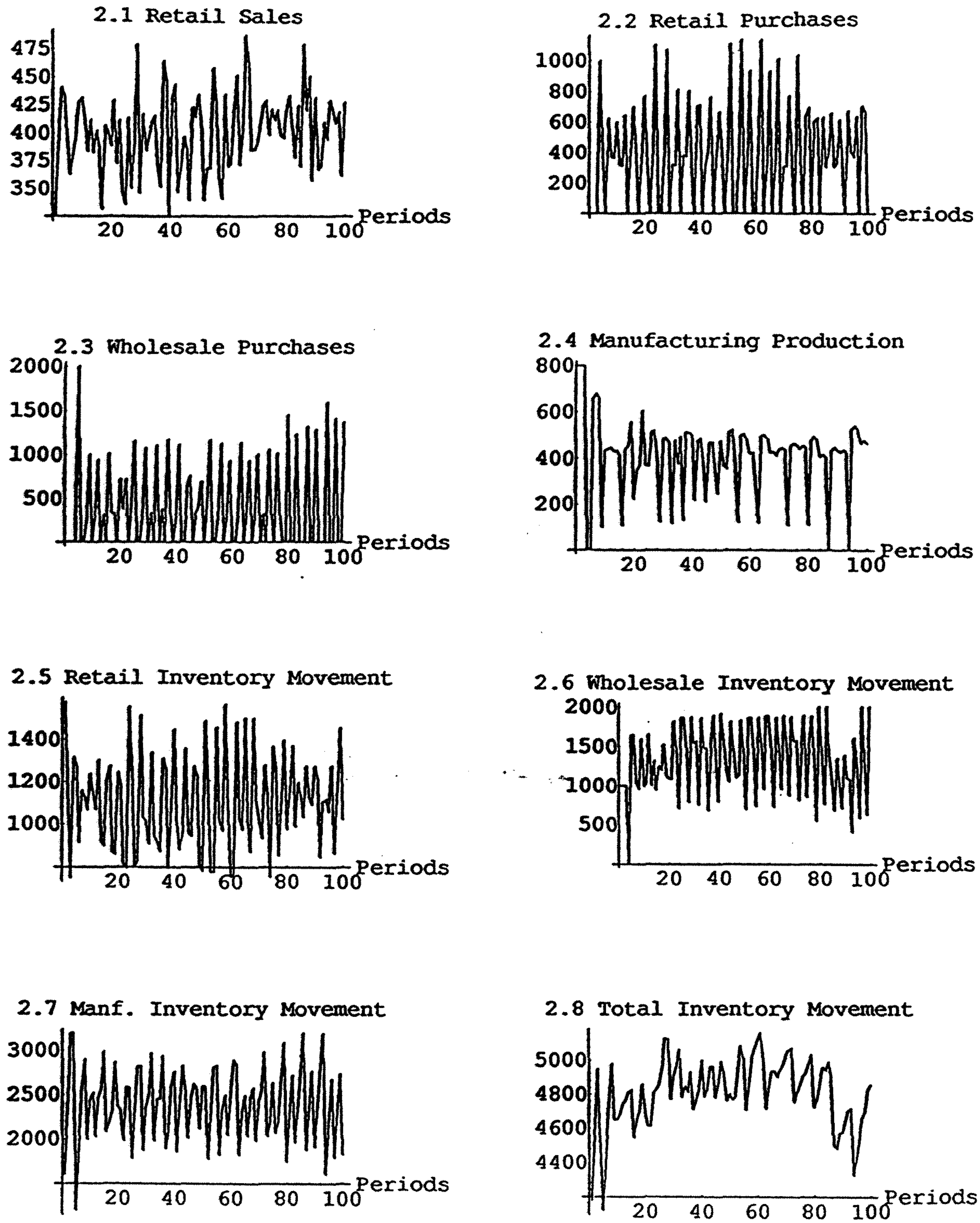


Figure A3 Sales, Purchases and Inventory for Case 3

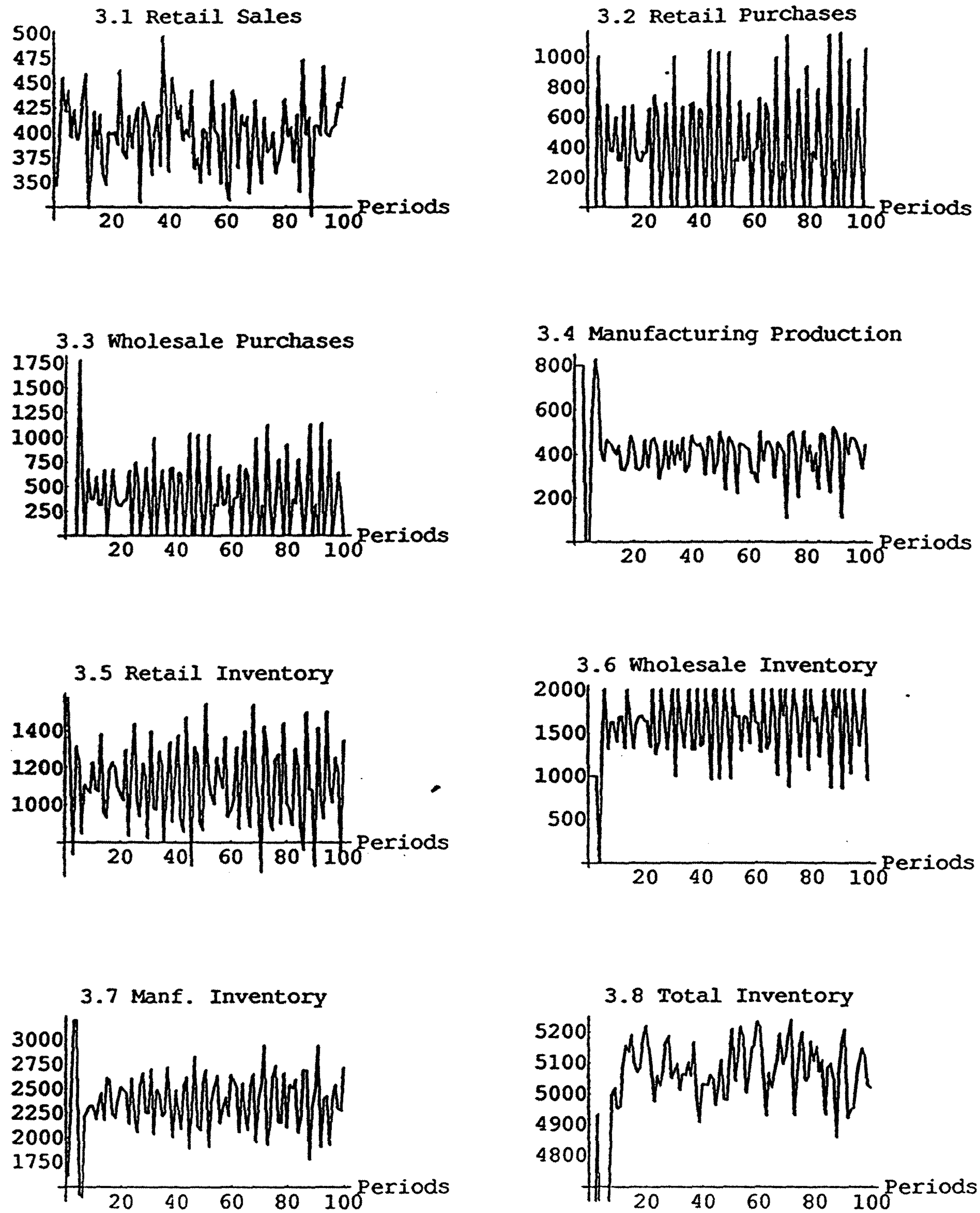


Figure A4 Sales, Purchases and Inventory for Case 4

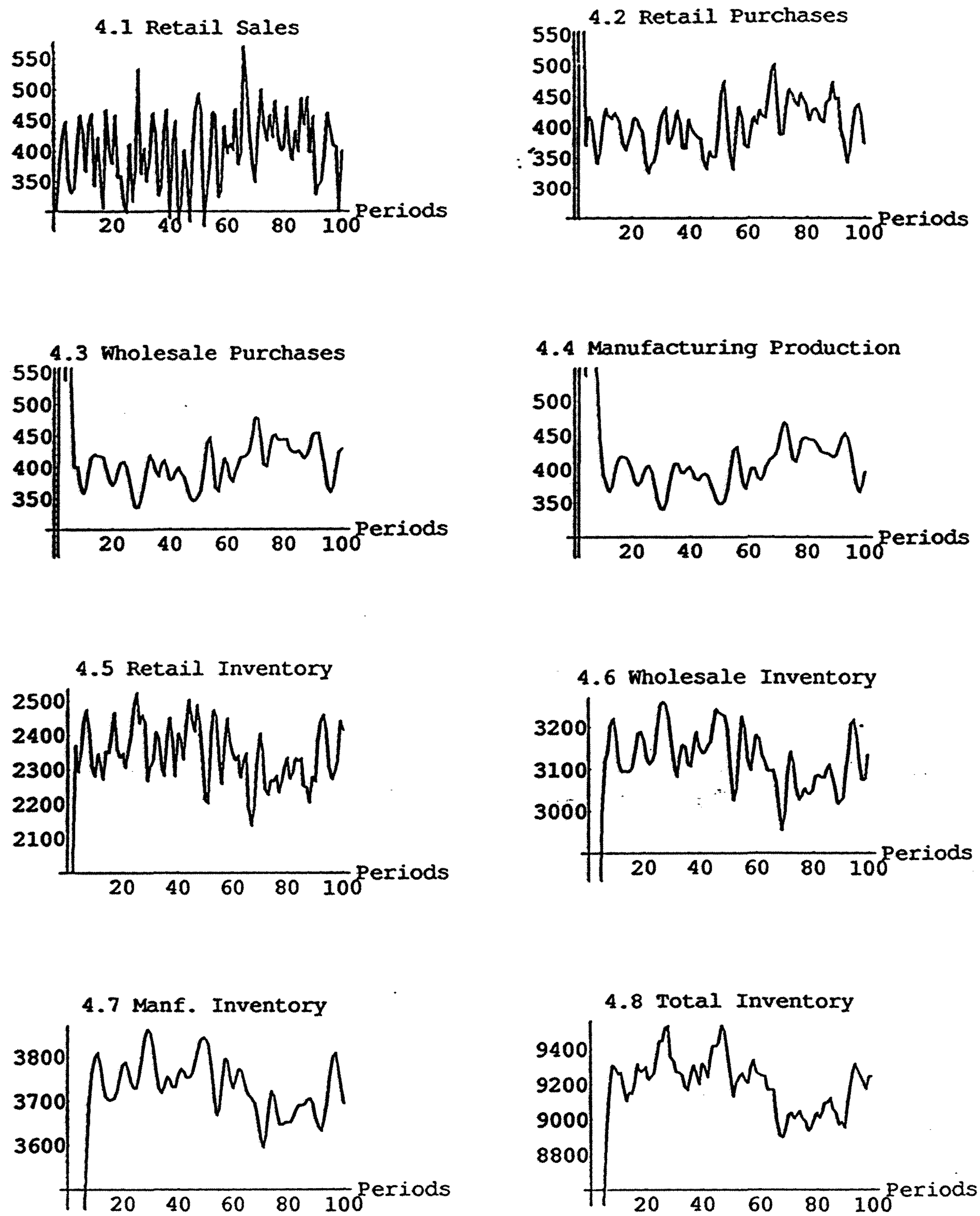


Figure A5 Sales, Purchases and Inventory for Case 5

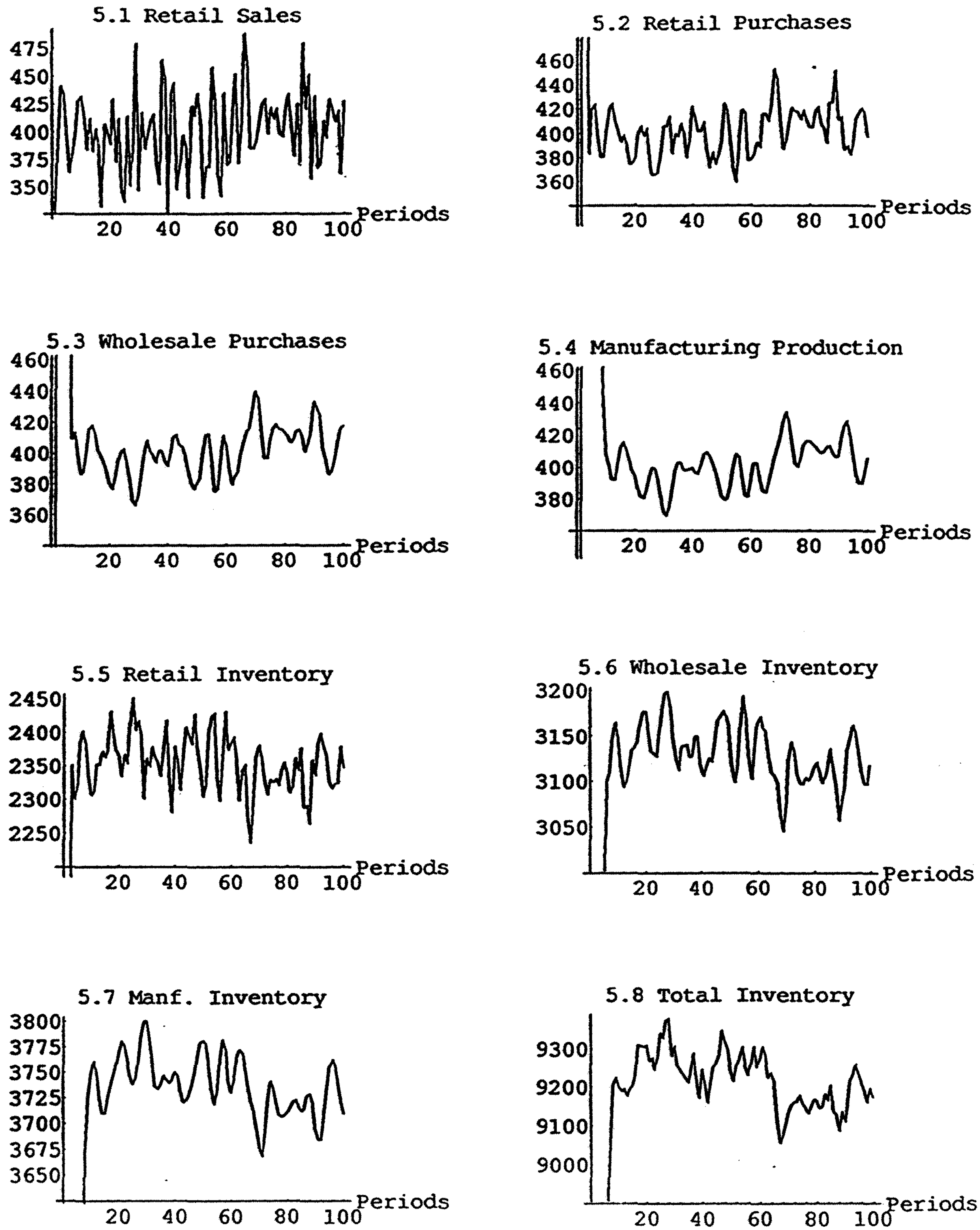


Figure A6 Sales, Purchases and Inventory for Case 6

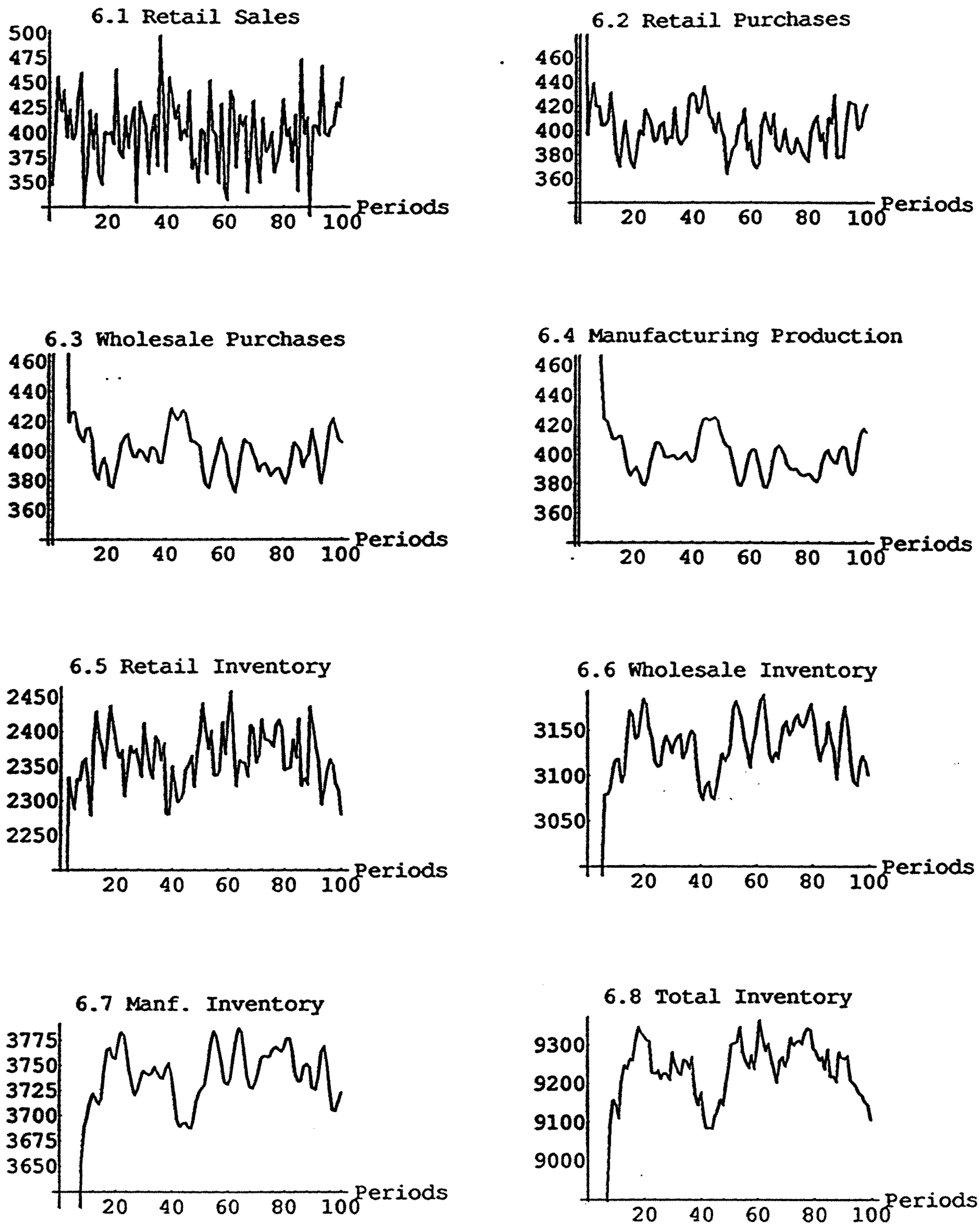
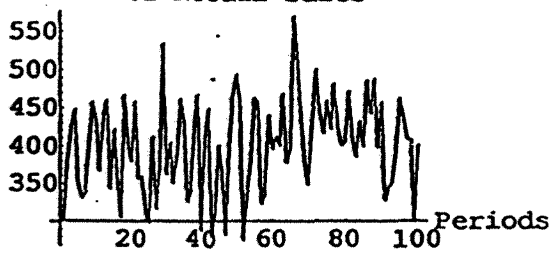
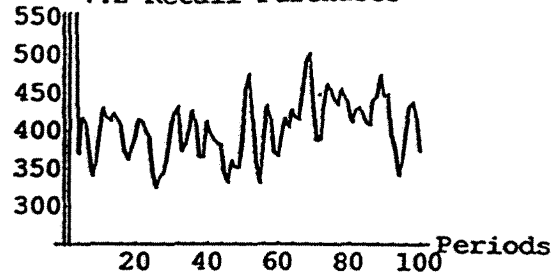


Figure A7 Sales, Purchases and Inventory for Case 7

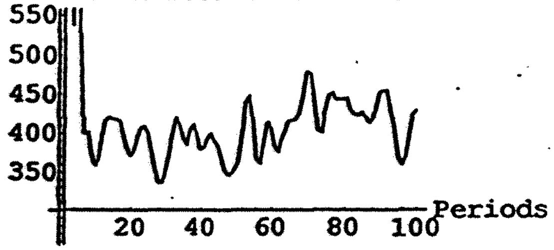
7.1 Retail Sales



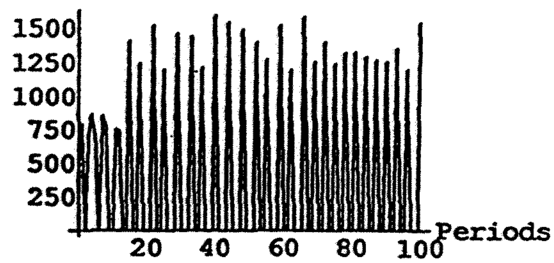
7.2 Retail Purchases



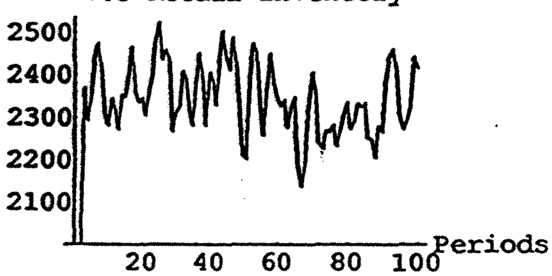
7.3 Wholesale Purchases



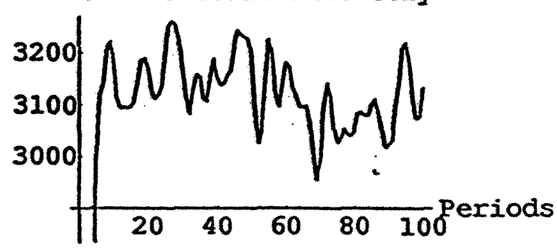
7.4 Manufacturing Production



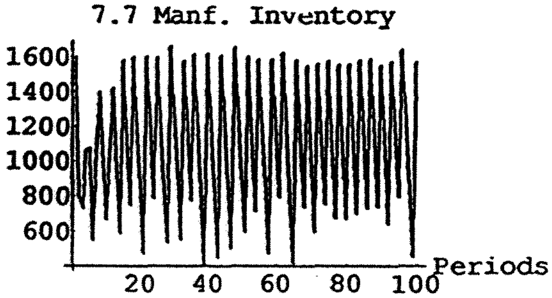
7.5 Retail Inventory



7.6 Wholesale Inventory



7.7 Manf. Inventory



7.8 Total Inventory

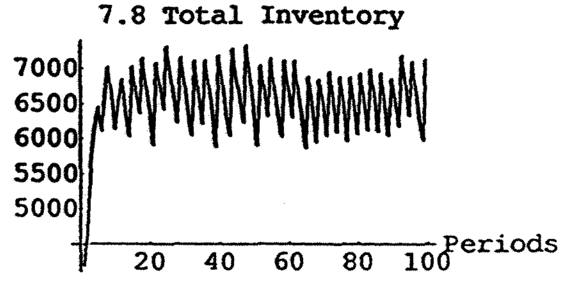


Figure A8 Sales, Purchases and Inventory for Case 8

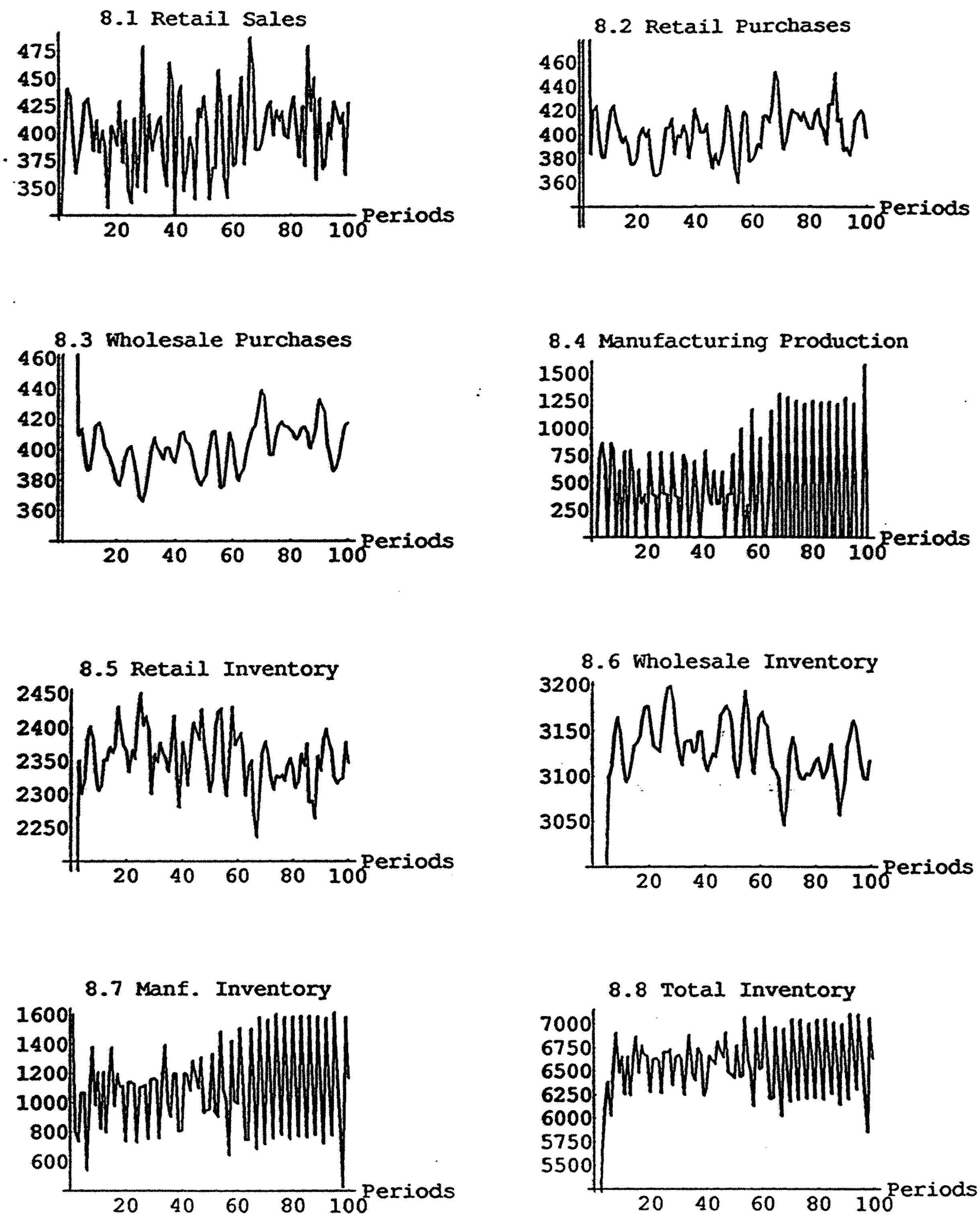


Figure A9 Sales, Purchases and Inventory for Case 9

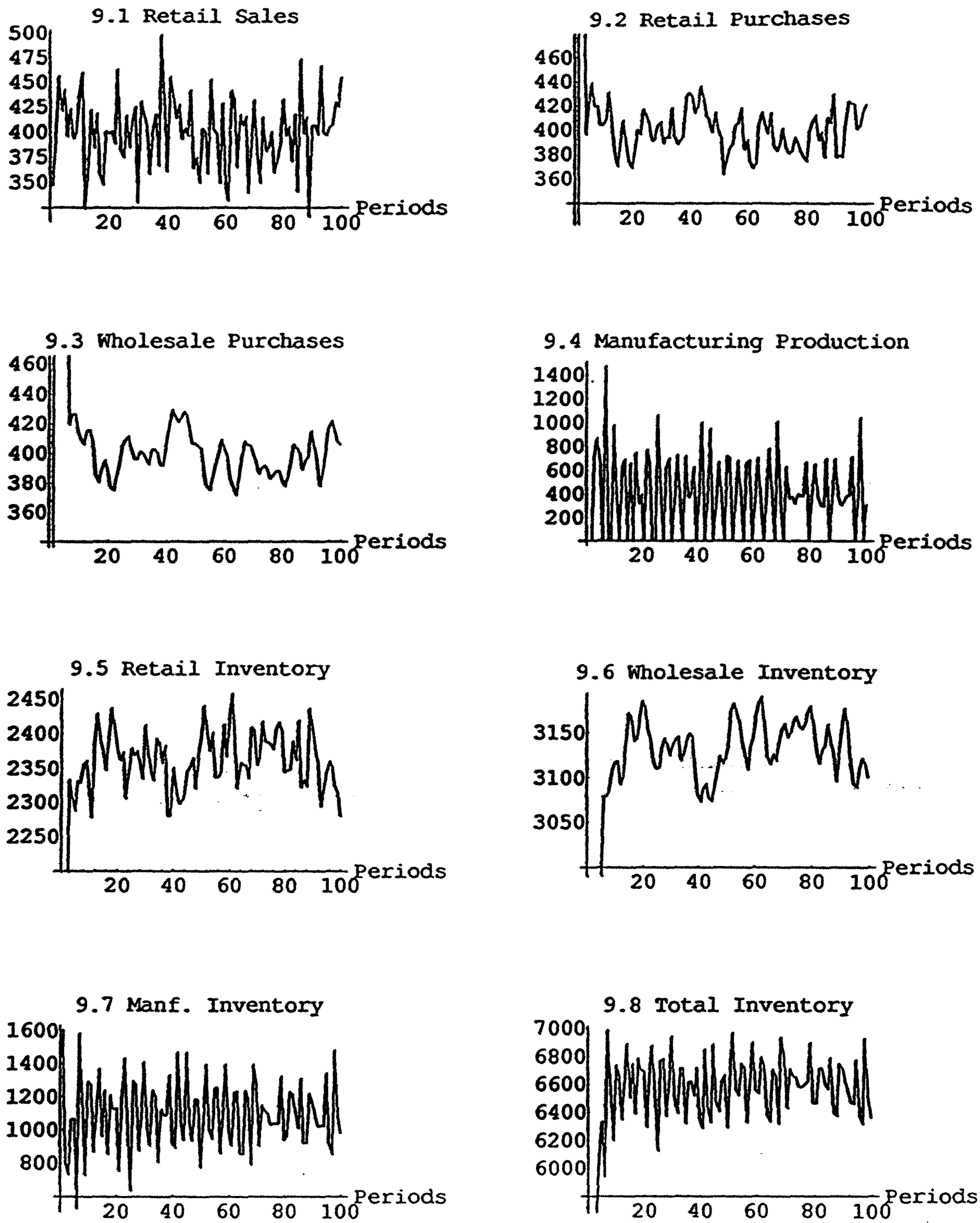


Figure A10 Sales, Purchases and Inventory for Case 10

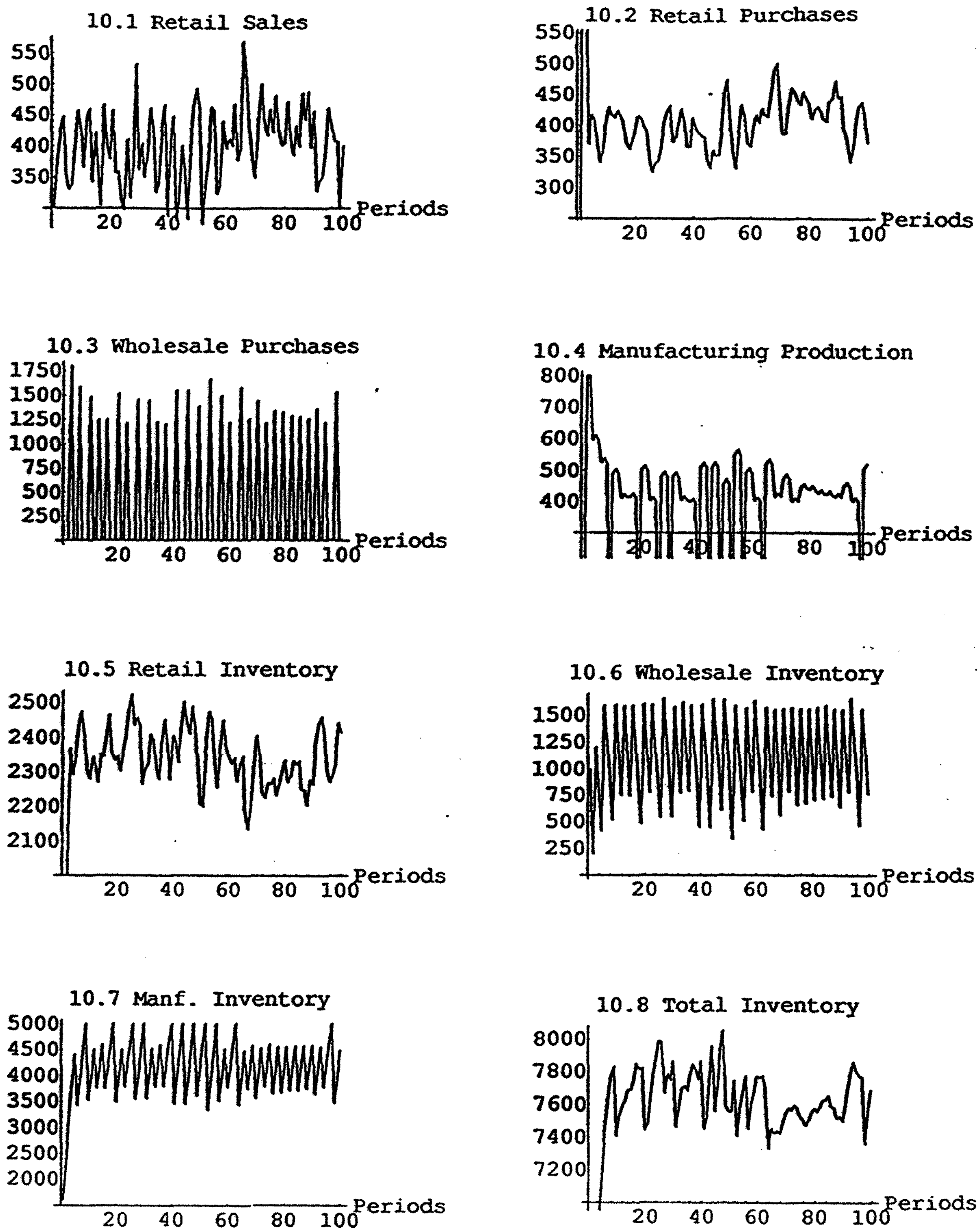


Figure A11 Sales, Purchases and Inventory for Case 11

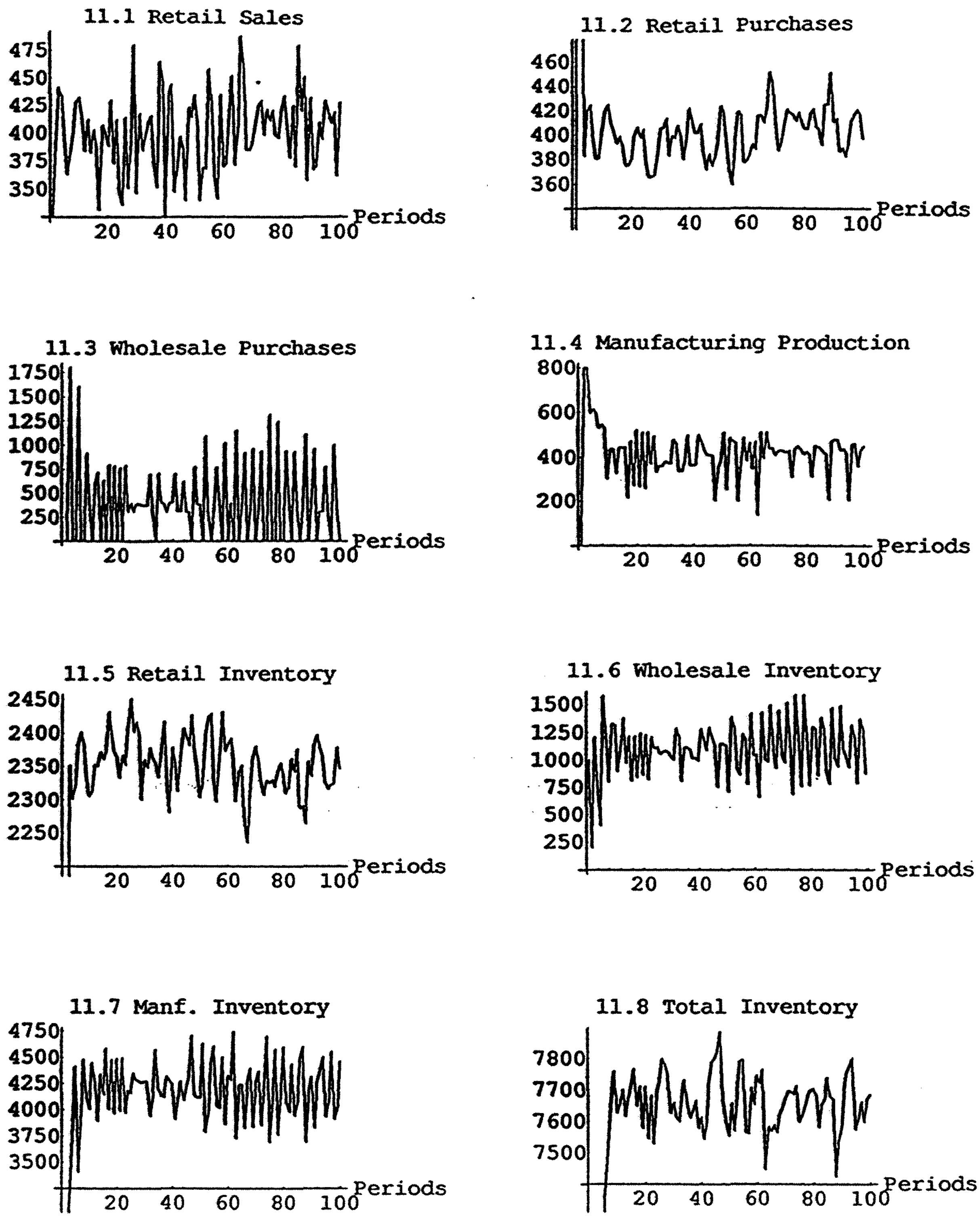


Figure A12 Sales, Purchases and Inventory for Case 12

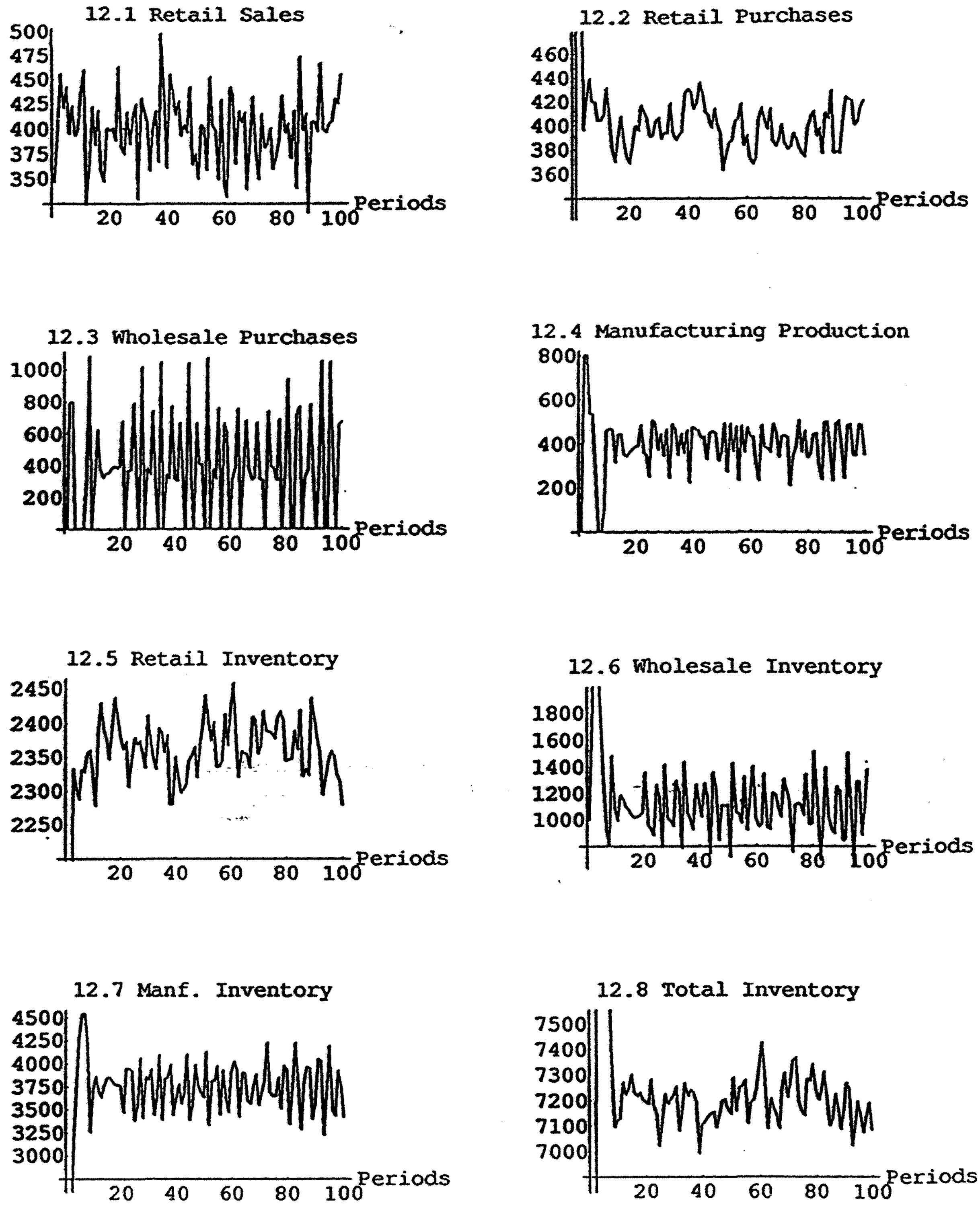


Figure A13 Sales, Purchases and Inventory for Case 13

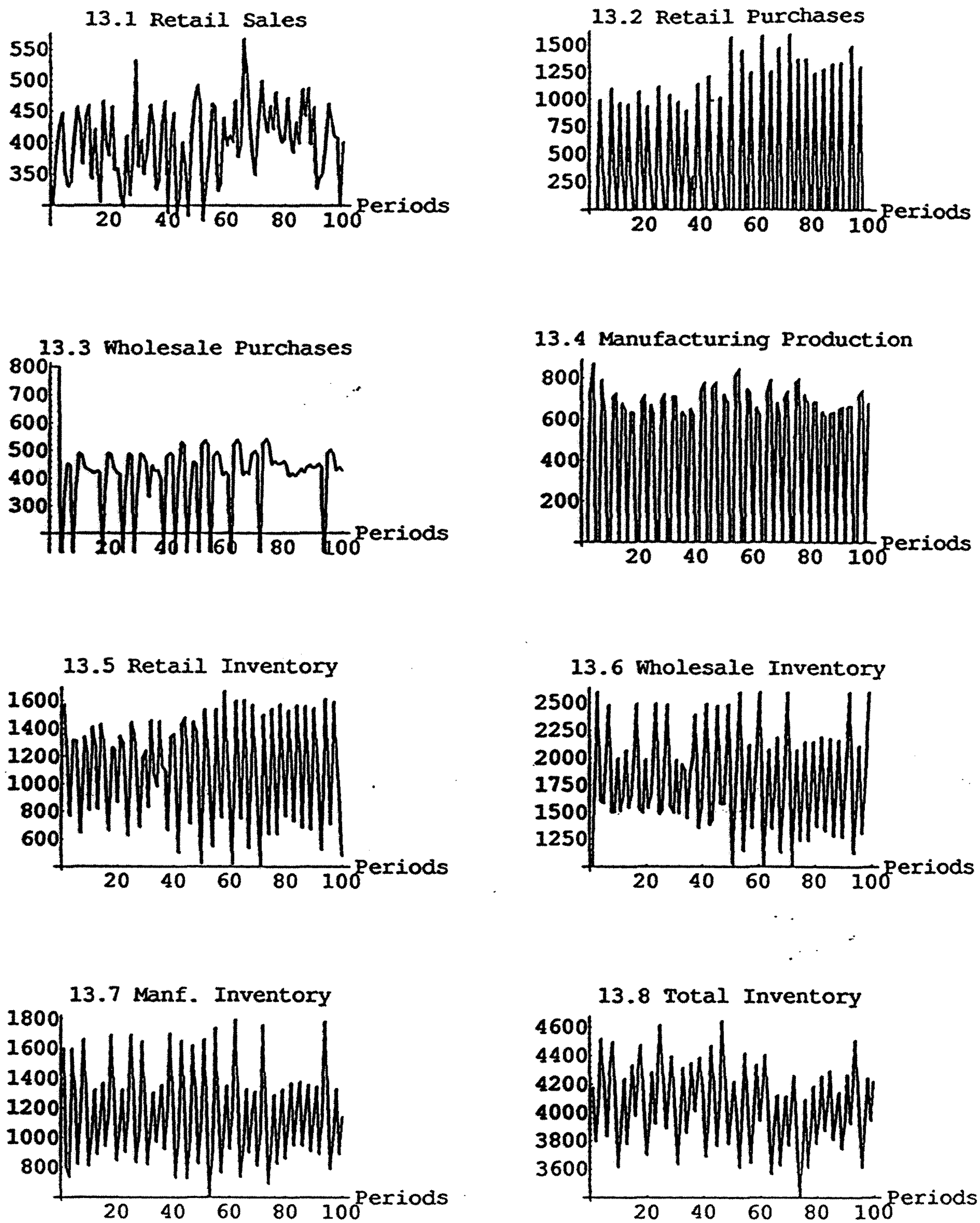


Figure A14 Sales, Purchases and Inventory for Case 14

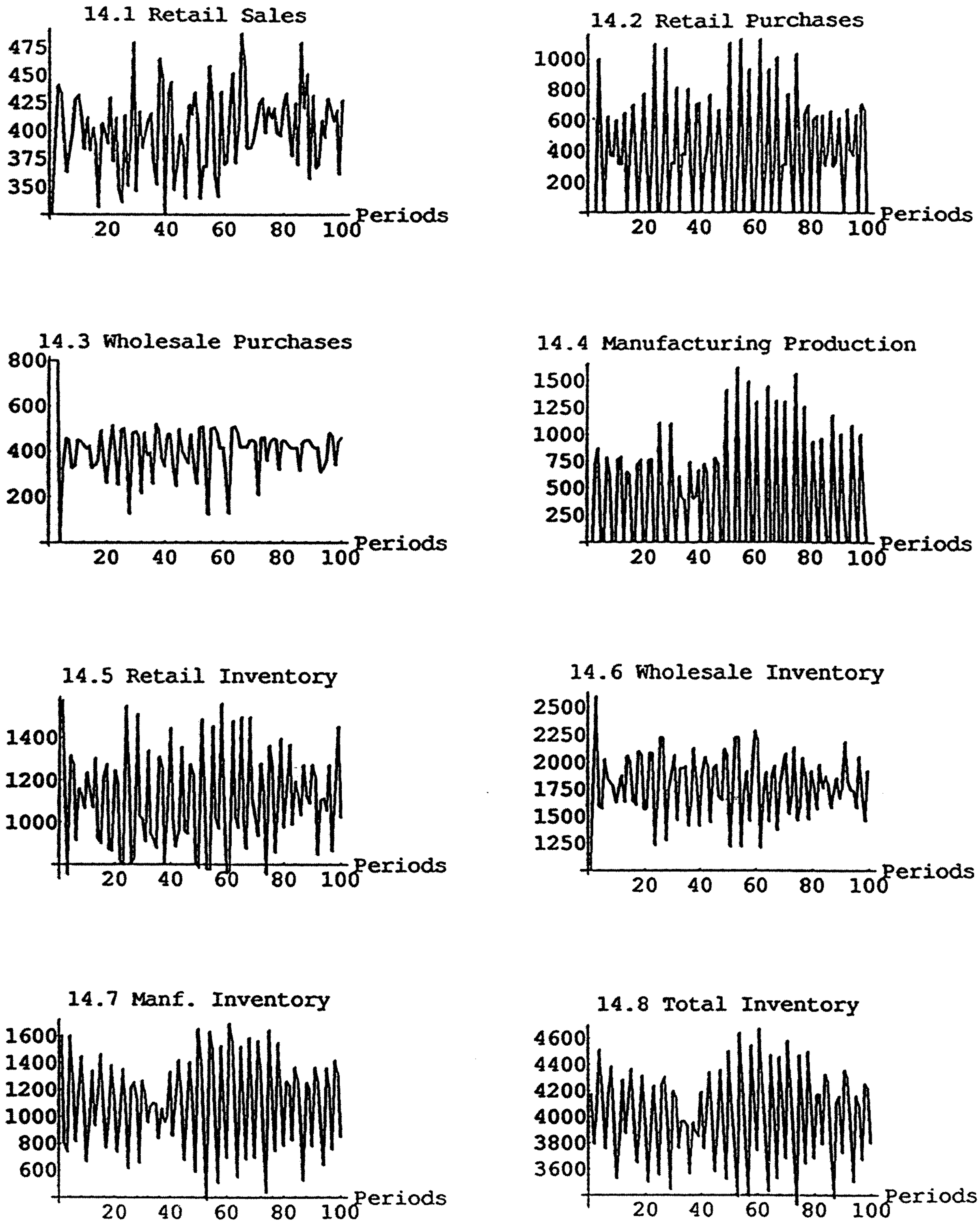
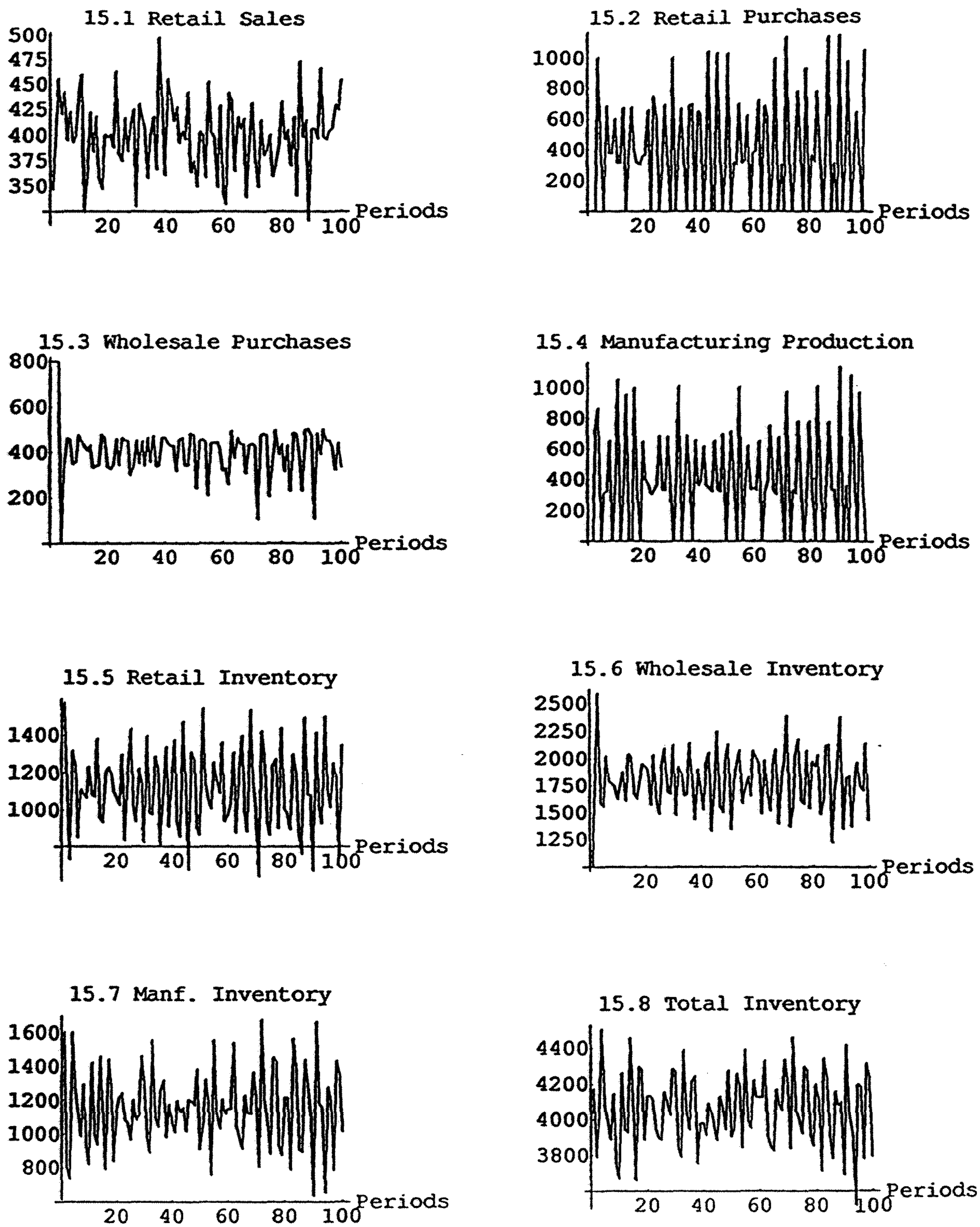


Figure A15 Sales, Purchases and Inventory for Case 15



REFERENCES

Allen, Donald S., *Inventory Investment Dynamics: A Heterogeneous Coupling Explanation*, Ph.D. Dissertation, Washington University, 1994.

Blinder, Alan S., *Inventory Theory and Consumer Behavior*, University of Michigan Press, Ann Arbor, 1990.

_____ and Louis J. Maccini, "The Resurgence of Inventory Research: What Have We Learned?", *Journal of Economic Surveys*, 1991, Vol. 5, No. 4, pp. 291-328.

_____ and Louis J. Maccini, "Taking Stock: A Critical Assessment of Recent Research on Inventories", *Journal of Economic Perspectives*, Winter 1991, Vol. 5, No. 1, pp 73-96.

Bertola, Giuseppe, and Ricardo J. Caballero, "Kinked Adjustment Costs and Aggregate Dynamics", *NBER Macroeconomics Annual*, 1990, pp. 237-295.

Bridge, J. L., *Applied Econometrics*, North-Holland, Amsterdam London, 1971.

Caballero, Ricardo J., and Eduardo M. R. A. Engel, "Dynamic (S,s) Economies", *Econometrica*, November, 1991, Vol. 59, No. 6, pp. 1659-1686.

Caplin, Andrew S., "The Variability of Aggregate Demand With (S,s) Inventory Policies", *Econometrica*, November, 1985, Vol. 53, No. 6, pp. 1395-1409.

_____ and Daniel F. Spulber, "Menu Costs and The Neutrality of Money", *Quarterly Journal of Economics*, November, 1987, Vol. CII, pp. 703-725.

Carpenter, Robert E., Steven M. Fazzari, Bruce C. Petersen, "Inventory (Dis)Investment, Internal Finance Fluctuations, and the Business Cycle", forthcoming *Brookings Papers on Economic Activities*, 1993.

Christiano, Lawrence, Martin Eichenbaum, "Temporal Aggregation and Structural Inference in Macroeconomics," *Carnegie-Rochester Conference Series in Public Policy*, Spring 1987, 26, 63-130.

Cooper, Russell, and John Haltiwanger, "Inventories and the Propagation of Sectoral Shocks", *The American Economic Review*, March, 1990, Vol. 80, No. 1, pp. 170-190.

Doebelin, Ernest O., *System Dynamics: Modeling and Response*, Charles E. Merrill Publishing Company, Columbus, Ohio, 1972.

Dorf, Richard C., *Modern Control Systems*, Addison Wesley, New York, 1989, 5th Edition.

Eichenbaum, Martin, "Some Empirical Evidence on the Production Level and Production Cost Smoothing Models of Inventory Investment", *American Economic Review*, 1989, Vol. 79, No. 4, pp. 853-864.

Fair, Ray C., "The Production Smoothing Model is Alive and Well", *Journal of Monetary Economics*, 1989, Vol. 24, pp. 3553-370.

Harvey, Andrew C., *The Econometric Analysis of Time Series*, The MIT Press, Cambridge, Massachusetts, 1990, 2nd Edition.

Hay, Donald, and Helen Louri, "Investment in Inventories: An Empirical Microeconomic Model of Firm Behavior", University of Oxford Working Paper, September, 1991.

Holt, Charles C., Franco Modigliani, John F. Muth, Herbert A. Simon, *Planning Production, Inventories, and Work Force*, Prentice - Hall, Inc., Englewood Cliffs, New Jersey, 1960.

Hordijk, Arie, and Frank A. Van Der Duyn Schouten, "On the Optimality of (s,S)- Policies in Continuous Review Inventory Models", *SIAM Journal of Applied Mathematics*, October 1986, Vol 46, No. 5, pp. 912-929.

Kahn, James A., "Inventories and the Volatility of Production", *The American Economic Review*, September, 1987, Vol. 77, No. 4, pp. 667-679.

Kashyap, Anil K., and David W. Wilcox, "Production and Inventory Control at the General Motors Corporation During the 1920's and 1930's", *The American Economic Review*, June 1993, Vol. 83, No. 3, pp.383-401.

Krane, Spenser D., "The Distinction Between Inventory Holding and Stockout Costs: Implications for Target Inventories, Asymmetric Adjustment, and the Effect of Aggregation on Production Smoothing", *International Economic Review*, Vol. 35, No. 1, February 1994, pp. 117-136.

_____ and Steven N. Braun, "Production Smoothing Evidence From Physical Product Data", *Journal of Political Economy*, June 1991, vol. 99, pp. 558-81.

Lai, Kon S., "Aggregation and Testing of the Production Smoothing Hypothesis", *International Economic Review*, Vol. 32, No. 2, May 1991, pp 391-403.

Lovell, Michael C., "Simulating the inventory cycle", *Journal of Economic Behavior and Organization*, 1993, pp. 147-179.

Lundberg, Erik, *Studies in the Theory of Economic Expansion*, P. S. King, London, 1937, chap. 9.

Mack, Donald R., *The General Electric A Course: Engineering Analysis*, General Electric Company Advertising and Sales Promotion Operations, Norwalk, Connecticut, 1982, Ch. 14.

Maccini, Louis J., and Robert J. Rossana, "Joint Production, Quasi-Fixed Factors of Production, and Investment in Finished Goods Inventories", *Journal of Money, Credit and Banking*, May 1984, Vol. XVI, No. 2, pp. 218-236.

Metzler, Lloyd A., "The Nature and Stability of Inventory Cycles", *Review of Economics and Statistics*, 1941, vol. 23, pp. 113-129.

Miron, Jeffrey A., and Stephen P. Zeldes, "Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories", *Econometrica*, July, 1988, Vol. 56, No. 4, pp. 877-908.

Morgan, Donald P., "Will Just-In-Time Inventory Techniques Dampen Recessions?" *Federal Reserve Bank of Kansas City Economic Review*, March/April 1991, pp. 21-32.

Mosser, Patricia C., "Trade Inventories and (S,s)", *The Quarterly Journal of Economics*, November, 1991, pp. 1267-1286.

Nakamura, Masao, and Alice Nakamura, "Inventory Management Behavior of American and Japanese Firms", *Journal of the Japanese and International Economies*, 1989, Vol. 3, pp. 270-291.

Ogata, Katsuhiko, *Modern Control Engineering*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970.

Ramey, Valerie A., "Nonconvex Costs and the Behavior of Inventories", *Journal of Political Economy*, 1991, Vol. 99, No. 2, pp. 306-334.

_____, "Inventories as Factors of Production and Economic Fluctuations", *American Economic Review*, June 1989, Vol. 79, pp. 338-354.

Ross, Sheldon M., *Applied Probability Models with Optimization Applications*, 1970, Dover Publications, Inc., New York, pp. 169-179.

Scarf, Herbert, "The Optimality of (S,s) Policies in the Dynamic Inventory Problem", *Mathematical Models of the Social Sciences*, K. J. Arrow, S. Karlin and P. Suppes eds., 1959, Stanford University Press.